Fair Payoffs Distribution in Linear Production Game by Shapley Value

Benjawan Intara

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Computer Science at Mahasarakham University

June 2022

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เบญจวรรณ อินทระ

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The examining committee has unanimously approved this dissertation, submitted by Miss. Benjawan Intara, as a partial fulfillment of the requirements for the Doctor of Philosophy in Computer Science at Mahasarakham University.

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ABSTRACT

Strategic form game (SFG) has been used widely to model inter-related decision making. Generally, researchers work on a particular game, specified by certain actions and corresponding payoffs. In real world, the situation can be much more complex, a particular game may not be enough. Furthermore, the actions and payoffs are not known a priori. Here, we consider a more realistic environment, where payoffs are to be optimally computed from given resources and be used by agent for making decision. We are interested in wider spectrum of outcomes in games, where payoffs can vary within a trend such that the agents' strategies remain unchanged. The results show that there exist certain ranges of resources that agents do not change there strategies. Hence, agents receive fair payoffs. Furthermore, taking into account additional computations normally take place in real world environment do not affect the acceptable computation time for agents payoffs.

Keywords : Fair Payoff, Non-cooperative game, Payoff Trends.

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CHAPTER 1

INTRODUCTION

Game theory has been an important area of research in multiagent systems because it give underpinning principles for interactions, regardless of competition or cooperation, among agents. While competition, known as *non-cooperation*, has received a lot of attention in the past decades due the great success of Nash in winning the so called Nobel Prize in Economics, cooperation, known as *coalition formation* has gained more attention recently. Two of cooperative game theorists have won the same award twice in 2005 and 2012 respectively.

While non-cooperative games concentrate on how to react wisely to other agents' strategies, coalition formation concentrate on how to distribute the benefit received from the cooperation among agents. The distributed benefits to agents are referred to as *payoffs*. Many solution concepts for distributing the benefit to agent were proposed, including the efficiency, stability and fairness. Efficiency means that the payoffs for agents are efficient. Stability means that the payoffs do not allow any agent to deviate from their group for higher payoffs. Fairness means that the payoffs was calculated fairly.

The solution concept for fairness is known as *Shapley Value*. It was proposed by Loyd Shapley, the winner of Nobel Prize of Economics in 2012. It has been regarded highly because it provides the agents fairness, based on their contributions to the coalitions' values. However, the concept works in the *superadditive* environment, one in which the value of a coalition is not less than that if its constituting coalitions. It has been shown that we cannot always assume superadditive environment because larger coalitions may incur some costs hence reducing the values. Such environment is referred to as *non-superadditive*. Another complex environment is externalty, one in which strategies of agents outside a coalition affects its value.

In general, this research investigates the behavioral results of applying Shapley value in multiple settings. Most interestingly, we propose a simple game, namely *Bakery Game*, in which cooperation does not guarantee higher coalition values. The total contributions are in the following. In Chapter 3, we investigate fair payoff division in characteristic function game both in super additive and non super additive. In Chapter 4, we investigate fair payoff division in Linear production game where we explore relation ship between value and quantity of resources passed by agents and their payoffs in cooperative environment. In Chapter 5, we investigate fair payoff division in Bakery Game where we explore relationship between value quantity of resources against payoff and strategy of agent in non cooperative environment. In Chapter 6, we investigate fair payoff division in generic non-cooperative strategic form game where we explore relationship between agents payoff and their strategies. Fair can lead to prediction of outcome of games and in reverse design the rule of the game which will yield desired outcome.

1.1 Objective

This research is to

1. investigate the payoffs for agents under the domain of study.
2. investigate the behavioral results (trends) of applying Shapley value in non-superadditive environment with externalties. We propose a simple game, namely Bakery Game, in which cooperation does not guarantee higher coalition values.
3. propose a compensation scheme for agents to help secure the efficiency of the system.

1.2 Scope

This research will be conducted under the environment below:

1. The number of agents is 15. The complexity of computing Shapley value is exponential due to the nature of permutation ($n!$), while computing agents' contribution. In practice, computing permutation on typical computers can take up to around 15 agents within reasonable time.
2. The domain of problem is the class of linear production game, namely bakery game. This game is more complex than the environment Shapley value was

originally designed. Here, the strategies of agents outside a coalitions affects its value. In this bakery game, the higher number of goods produced into the market, the lower the unit price, and eventually the lower the profit.

1.3 Terminology

1. Characteristic function, function of determine coalition value of agent.
2. Coalition, interest centers on when several players together make a binding agreement to coordinate their efforts in a joint decision which, perhaps, might not be guaranteed if they acted separately.
3. Coalition Structure (CS) is means of describing how the players in N divide themselves into mutually exclusive and exhaustive coalitions.
4. Coalition Value, value of agent in coalition.
5. Payoff, the quantitative representation to a player of an outcome.
6. Contribution, value of agent that agent can contribute in coalition.

CHAPTER 2

LITERATURE REVIEW

In real world, many forms of cooperation, including Shapley value, have been used extensively throughout the years on several domains. Below, we review the recent works of applying the concepts as shown in Figure. 2.1.

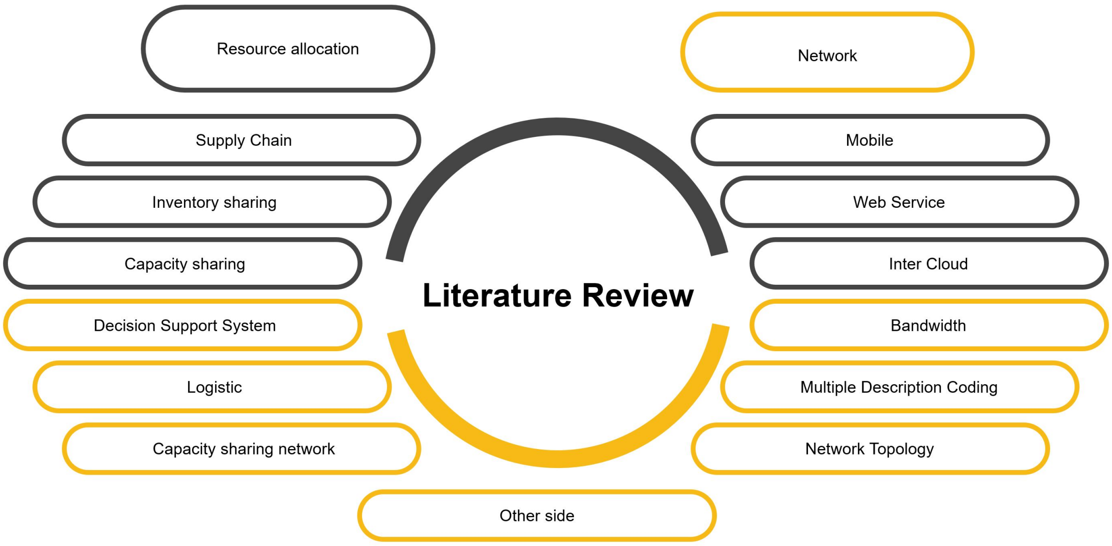


Figure. 2.1. Overview of literature review.

2.1 Resource allocation

In general, resource allocation aims at maximizing profit. Once there are multiple agents pooling resources, the profit accruing from cooperative agents are to be fairly distributed among themselves by using Shapley value.

2.1.1 Supply Chain

The first domain we look at is in supply chain. The original Shapley value is extended to work in uncertain environment into two dimensions , namely expected Shapley value and $\alpha-$optimistic Shapley value [15]. They are used to help allocation profits for supply chain alliance. Since the complexity of computing Shapley value is NP-Hard, efficiency in using computational power is very important. For the computing algorithm, this can be implied that it must estimate the fair shares for agents as quickly as possible. Bhagat et al. [9] introduce a framework that can work with a wide variety of game with efficient algorithm. Collaboration among multiple parties can be complex because disruptions always take place. This needs appropriat protocol to help make multi-sourcing decision. Seoket et al. [45] introduce the Intelligent Contingent Sourcing (ICS) protocol, using Shapley value concept to help reach conclusion quickly.

2.1.2 Inventory and Capacity sharing

Most businesses require proper capacity in their inventory in order to keep with fluctuation in their supplies. However, large amount of supplies stocking requires large capacity in their inventory. However this may eventually cost them expensively because of uncertainty in demands. It is recommended that sharing information among retailers can solve this problem. This can be achieved by a proper coordination mechanism [61], based on Shapley value. Since computing Shapley value is NP-hard, it is often that we estimate the value, instead. For a collaboration of upto 100 agents, greedy heuristic algorithm is used to estimate and achieve satisfactory results, within 0.12% of the optimal solution [21]. To work with decentralized dealer network, Zhao et al. [65] analyze the problem and propose a model for inventory sharing. The sharing behaviors are classified into multiple classes. The principle of computing Shapley value is applied to achieve satisfactory results.

Collaboration among a set of enterprises incurs many challenges. Among these, communication protocols is very important because it can bring disscussion issues to conclusion rapidly. Yoon et al. [64] propose a Shapley-based protocol for sharing their demands and capacities, helping increase demand fulfillment rate and total profit. Shippling industry also benefit from the concept of Shapley value. One of the outstanding problems is about settling their account balances. Li et al. [31] use a Shpaly-based model to help the shipping forwarders to purchase shipping capacity from each other. after they order capacity from the carrier but before they set the selling prices. Yoon et al. [63] designed to find efficient demand and capacity sharing decisions in the CN, by the proposed demand and capacity sharing decisions and protocols can significantly increase the demand fulfillment rate and the total profit of the CN. Seok et al. [46] design the Adaptive CDCS(Collaborative Demand and Capacity Sharing) protocol based on dynamic contract mechanism. To deal with volatile product demand and rapidly changing manufacturing technologies for sustainable returns. Lastly, Renna et al. [43] help reduce capacity investment value with a Shapley-based model.

2.1.3 Decision Support System

Decision support system is also benefited from Shapley value. Schleich et al. [44] use an algorithm that helps balance inventorry and demand sharing, using deficit satisfaction, efficientlyl Andres et al. [3] introduce a Shapley value-based principle to select strategies with higher alignment levels, resulting in successful collaborations among enterprises.

2.1.4 Logistics Domain

Logistics is a very challenging but important domain. Based on the belief that adequate cost allocation can increase efficiency in transforming cost allocation among logistic partners [13]. Complexity in logistics demand a lot of computational power. Kimms et al. [24] proposed a solution procedure to relax this demanding stress, resulting in shorter time computation. Defryn et al. [12] optimize different demands of multiple companies with their own set of objectives, resulting good quality solutions. Renna et al. [42] proposes a Shapley-based mechanism for coordination in a network of independent plants, resulting in increasing performances in various environmental conditions.

2.2 Network Domain

Among many domains, Shapley value is extensively used to help distribute packets of data from differnt sources towards their destination fairly. Althought the most important aspect of computer networking is to maximize throughput of the network, distributing packets fairly help increase the network throughput because there is unlikely dominant users in the network that keep other users waiting for their turns to send data.

2.2.1 Mobile Network and Wireless Network

The first network application is mobile phone. Shapley value can be used to provide the overall best performance in terms of throughput, fairness, and transmit power

[28]. However, there can be a slightly higher time complexity. It is recommended that the cost savings should be dispersed among the cooperating service provider [25] by applying a game-theoretic framework with axiomatic Shapley value rule. Shapley value concept can be used in crowd sensing by integrating integrate quality estimation incentives and surplus sharing Yang et al. [62]. It is shown that Shapley value can be used to help increase performance of cooperative communication between relays and base station in MIMO-OFDM framework [7].

The next domain is wireless network. It is shown that coupling heterogeneous wireless networks (HetNets) and multiple clouds can provide effective response for the mobile data in cloud computing (MCC) environment [11]. Shapley value can also be used to provide security in the network as well [58].

2.2.2 Web Service

Another sub-domain is web service. Shapley value is used as the principle algorithm for providing an efficient community formation mechanism [5]. The experimental results provide near-optimal decision making mechanisms. Mong et al. [57] theoretically prove that these inter-cloud coalition formation strategies can help reach subgame perfect equilibrium. Furthermore, it is shown that inter-cloud coalition can achieve fair payoffs. Shapley value is extended to a one-shot auction algorithm. Users submit their bids at the same time. This help improve bandwid performance in the network [2]. Cooperation among internet service providers (IPS) are an important factor to performance of the internet. It is proved that IPSs of different regimes on the traffic demand and network bandwidth can cooperate based on Shapley value for better performance [30]. In inter data center on demand bandwidth, a Shapley value based auction is used to achieve first dynamic pricing mechanism that help improve performance [56]. In addition, a Shapley value based auction can improve throughput in the network [6].

2.2.3 Network Topology

Designing network topology is a significant factor for network performance. Apostolaras et al. [4] simulate the operation of the LTE-A network, and conducting test bed experiments for the mesh network. It is found that it can achieve significant savings for eNBs power consumption and reimbursements for mesh users. Oikonomakou et al. [37] can achieve significant efficiency of energy by switching off scheme by bankruptcy game. This provide a balanced and satisfactory cost allocation for different MNO traffic loads. Muros et al. [34] apply Shapley value to improve quantifying the value of the communication under different control topologies. Sharma et al. [50] achieve stable solution that combine appropriate penalties or rewards to participants.

2.3 Other Domains

In addition to resource allocations and network, we also review Shapley value in other domains. The first one to be discussed in in energy industry. Global optimization is needed for energy saving. This can be achieved by optimizing timetable and speed profile. This can be achieved by formulating an integrated energy-efficient timetable and speed profile optimization model [32]. The concept of fairness is also used in pricing by estate dealer [19]. An optimal dealer pricing model under transaction uncertainty is used to maximize the dealer's total wealth, Balancing work force for fluctuating need is a tough task. We do not know in advance for certain when the exactly the actual need for work force will take place. However, holding the workforce in permanent positions can be costly waste. One approach to provide certainty for this issue is to apply Branch-and-Price approach to find a stable workforce assignment [14] To help establishing business partners network, Baum et al. [8] state that variation in age and size affect firm performance. A general equilibrium model for multiple firms is proposed by Nocke et al. [36]. The model compares some static information to decide whether firms differ in capabilities. This will eventually be used to calculate equilibrium. To create value chains across multiple firms [29], it is important that exploring and exploiting tendencies must be maintained over time. In addition, conflicting pressures may arise because of absorptive capacity and organizational inertia.

In hospital industry, it is important to balance between cost and quality to help attract income [60]. Imbalance between these two can be fatal to the success of this business. Similarly, construction industry need to ensure their quality of work (labor) much match with cost [41]. It is found that a mismatch between the actual role and position of their workforce can be a fatal factor for the success of the business. Balancing quality of service and cost is also a fatal factor for service industry [20]. This can be achievd by measuring and comparing service provider performance. the performance of physicians can be improved by detailing efforts based on the results of performance measurement. The fairness concept proposed by Shapley value can generally be applied for the aforementioned problems. Note that balancing different factors can also bring equilibrium to real world problem [1] that can be modeled as a non-cooperative strategic game problem.

2.4 Relation to Our Work

It has been shown that the principle of computing Shapley value has been widely adopted and extended to real world applications for long time. In this research, we further investigate the results of applying the principle of Shapley value in wider domains. both in theoretical basis and real world application basis.

CHAPTER 3

SHAPLEY VALUE IN CHARACTERISTIC FUNCTION GAME

In cooperative game, agent payoffs are the key factor to the outcome of the game, i.e., which coalitions will be formed and what payoffs the agents will achieved. The solution concepts provide different principles to consider the outcome of the games. Among many, efficiency is the most often studied concept. However, we suspect that achieving efficiency for the system may cause some agents their final payoffs which may not be individually maximal. This research seek to find if there are sacrificing agents for their global efficiency. And if that is the case how costly such sacrifice will be. We measure this by using their payoff value.

3.1 Coalition Formation

In contrast to non-cooperative game theory, cooperative game theory allows for agents to communicate that leads them to cooperation [26] from which they can benefit more individually. Agents communicate in order to negotiate with regard to whom they can cooperate and how the joint benefits will be distributed among them. When several agents make a binding agreement to cooperate, we say a coalition has been formed. Hence, the cooperative game theories are also known as the theories of *coalition formation* [26].

3.1.1 Coalition

Given $A = \{a_1, a_2, ..., a_{\mathbf{m}}\}$ a set of **m** agents, a *coalition* is a non-empty subset S of $A, S \subseteq A, S \neq \emptyset$. The set A itself is called the grand coalition while a coalition of one agent is called singleton coalition. Let S be the set of all coalitions, whose size of S is $(2^{\mathbf{m}} - 1)$. Given a set of 3 agents,

$$A = \{a_1, a_2, a_3\},$$

all the 7 coalitions are

$$\{a_1\}, \{a_2\}, \{a_3\},$$

$$\{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}$$

$$\text{and } \{a_1, a_2, a_3\}.$$

As in set theory, the cardinality, $|S|$, of S is the size of (the number of agents in) S.

3.1.2 Coalition Structure

Once agents have formed coalitions, they can be viewed as if they have divided themselves into a mutually exclusive and exhaustive partitions. We define a $coalition$ $structure$, CS, as a partition of M. A CS can be described by

$$CS = \{S_1, S_2, ..., S_\mathbf{m}\}.$$

The set of all CS is denoted by CS. For example, given $A = \{a_1, a_2, a_3\},$ all CS in S are

$$\{\{a_1\}, \{a_2\}, \{a_3\}\},$$

$$\{\{a_1, a_2\}, \{a_3\}\}, \{\{a_1, a_3\}, \{a_1\}\}, \{\{a_2, a_3\}, \{a_1\}\},$$

$$\{\{a_1\}, \{a_2\}, \{a_3\}\}.$$

A CS has to satisfy three conditions [26]:

1. $S_j \neq \emptyset, j = a_1, a_2, ..., a_M,$
2. $S_i \cap S_j = \emptyset$ for all $i \neq j$, and
3. $\cup$ S_j= M.

3.1.3 Coalition Value

The joint benefit of a coalition is call the *coalition value*, which is a numeric value that usually represents the utility which accrues from their cooperation. There is a characteristic function [26], υ that assigns a real number to each $S, \upsilon : 2^M \rightarrow \mathbb{R}$ We shall denote the coalition value of S with υ_S. Hence, a cooperative n-person game in characteristic function form is defined by the pair $(M; \upsilon)$ [26]. For example, a game is given with a characteristic function as shown below:

$\upsilon(\{a_1\}) = 2, \upsilon(\{a_2\}) = 4, \upsilon(\{a_3\}) = 3$
$\upsilon(\{a_1, a_2\}) = 5, \upsilon(\{a_1, a_3\}) = 6$
$\upsilon(\{a_2, a_3\}) = 8$
$\upsilon(\{a_1, a_2, a_3\}) = 10$

3.2 Optimal Coalition Structure (OCS)

Given a CS, we define its value,

$$\upsilon(CS) = \sum_{S \in CS} \upsilon_s,$$

An optimal coalition structure is a CS^* such that

$$\upsilon(CS)^* = argmax \sum_{S \in CS} \upsilon_s$$

For example:
$\upsilon(\{a_1\}, \{a_2\}, \{a_3\}) = 2 + 4 + 3 = 9,$
$\upsilon(\{a_1, a_2\}, \{a_3\}) = 5 + 3 = 8,$
$\upsilon(\{a_1, a_3\}, \{a_2\}) = 6 + 4 = 10,$
$\upsilon(\{a_1\}, \{a_2, a_3\}) = 2 + 8 = 10,$
$\upsilon(\{a_1, a_2, a_3\}) = 10$
$(CS)^* = (\{a_1, a_3\}, \{a_2\}), (\{a_1\}, \{a_2, a_3\})$ and $(\{a_1, a_2, a_3\})$
$\upsilon(CS)^* = 10$

3.3 Example of Cooperative Game

We now consider the classic Sandal Maker game [22, 47], in which there are five agents (sandal makers). Agent a_1 and a_2 make only left sandals, while agents $a_3, a_4,$ and a_5 make right sandals. In one cycle, a left sandal maker can produce 17 sandals, while a right sandal maker can produce 10 sandals. A single sandal is worth nothing, only a pair of left and right sandals can be sold for 20 dollars. Obviously, agents need to form coalitions of left and right sandal makers. Since sandals can only be sold in pairs, a singleton coalition or a coalition of the same side sandal makers (left or right only) cannot sell sandals. Hence, the coalition values are 0. A coalition of agents capable of producing both left and right sandals will be constrained by the smallest number of either side. Below is the characteristic function that summarizes coalition values as shown in table. 3.1:

Table. 3.1. coalition values

Coalitions	Value
$\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_5\}$	0
$\{a_1, a_2\}, \{a_3, a_4\}, \{a_3, a_5\}, \{a_4, a_5\}, \{a_3, a_4, a_5\}$	0
$\{a_1, a_3\}, \{a_1, a_4\}, \{a_1, a_5\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_1, a_5\}$	200
$\{a_1, a_3, a_4\}, \{a_1, a_3, a_5\}, \{a_1, a_4, a_5\}, \{a_2, a_3, a_4\}, \{a_2, a_3, a_5\}, \{a_2, a_4, a_5\}$	340
$\{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}, \{a_1, a_2, a_5\}$	200
$\{a_1, a_3, a_4, a_5\}, \{a_2, a_3, a_4, a_5\}$	340
$\{a_1, a_2, a_3, a_4\}, \{a_1, a_2, a_3, a_5\}, \{a_1, a_2, a_4, a_5\}$	400
$\{a_1, a_2, a_3, a_4, a_5\}$	600

If agent a_1 and a_3 agree to make a deal, while player a_2 and a_4 agree on another deal, a payoff configuration could be $(100, 50, 100, 150, 0; \{a_1, a_3\}, \{a_2, a_4\}, \{a_5\})$. Is this, however, a solution of the game? Since agents are self-interested, reaching such agreement may not always be this easy because there may be a chance that some agents are still looking to increase their payoffs. In the following, we shall explore solution concepts that bring stability to the game.

3.4 Games Environments

3.4.1 Superadditive

Classical research in cooperative game theory considers games within the *superadditive* [26] environment in which the value of a coalition is at least as much as the sum of the values of each pair of its subcoalitions, e.g.,

$$\upsilon_{(S \cup T)} \geq \upsilon_{(S)} + \upsilon_{(T)} \text{ for all } S, T \subseteq M \text{ such that } S \cap T = \emptyset.$$

In contrast to superadditive, the subadditive environment is one in which the coalition value of a given coalition is strictly less than the sums of the coalition value of each pair of its subcoalitions, e.g.,

$$v_{(S \cup T)} < v_{(S)} + v_{(T)} \text{ for all } S, T \subseteq M \text{ such that } S \cap T = \emptyset.$$

3.4.2 Non-Superadditive

In both environments, there is monotonicity in coalition value based on the size of coalitions. However, a *non-superadditive* [27] environment is one in which coalition values have no relationship to the size of coalitions at all. They are arbitrarily random. This environment is similar to the real world. It is less explored in cooperative game theory but has recently received more attention in multi-agent systems research recently multi-agent systems research recently [51, 54, 52, 10, 53, 55, 49].

3.5 Solution Concepts

Solution concepts are principle ideas that bring about stable state to the game. In general, agents are assume to be self-interested. They seek to maximize their payoffs. Agents choose the strategies that maximize their payoffs. There are principles of fairness, efficiency and stability. Shapley value ensures fairness for agents. The core enables efficiency for agents in the system. Kernel brings about stability to the system for agents. In this section we shall briefly discuss Kernel and the Core.

3.5.1 Kernel Solution Concept

Davis and Maschler [23] propose stabilty solution concept, namely, the *Kernel*. The kernel balances each pair of agents payoffs in each coalition. For a payoff configuration $(U; CS)$ of a given game, a group of agents M may leave their present coalitions to join a new coalition *R*.

The $excess : e(R; U) = v_R - U_R$ [23] is the difference between the value of R and the sums of their collective payoffs. For any two agents a_i, a_j in a coalition S, a_i may join R, and $a_j \notin R$. The largest of these excesses is called the maximum surplus of agent a_i over agent a_j with respect to $(U;CS)$, e.g.,

$$\mathfrak{S}_{a_i,a_j} = max_{R|a_i \in R, a_j \notin R} \qquad e(R;U).$$

Agent a_i can claim that it potentially could gain that much payoff. Similarly, agent a_j can do the same. There are three conditions that brings about stability: i) agent a_i has greater maximum surplus than a_j and can ask for compensation from agent a_j but not more than a_j's value, ii) agent a_j has greater maximum surplus than a_j and can ask for compensation from agent a_i but not more than a_i's value. iii) both a_i and a_j have the same maximum surplus. When one of these conditions are met for each pair of a_i and a_j for every coalition S in a given CS, we call that payoff configuration is in Kernel.

Let us consider the game [16],

$$\upsilon_{a_1} = \upsilon_{a_2} = \upsilon_{a_3} = 0;$$

$$\upsilon_{a_1,a_2} = 90; \upsilon_{a_1,a_3} = 80; \upsilon_{a_2,a_3} = 70;$$

$$\upsilon_{a_1,a_2,a_3} = 105.$$

Suppose we have a configuration payoff $(U;CS) = (45, 0, 35; \{a_1, a_3\}, \{a_2\})$ on offer. In order to find if it is in the Kernel, we find its excesses, maximum surpluses, then equilibrium as follows. Suppose agent a_1 and a_3 are considering their coalition $\{a_1, a_3\}$. There are two coalitions that include a_1 but exclude $a_3, e.g., \{a_1\}$ and $\{a_1, a_2\}$. The excesses and maximum surplus are

$$e(a_1; U) = \upsilon_{a_1} - U_{a_1} = 0 - 45 = -45.$$

$$e(\{a_1, a_2\}; U) = \upsilon_{a_1,a_2} - U_{a_1} = 90 - 45 = 45.$$

$$\mathfrak{S}_{a_1,a_3} = max(-45, 45) = 45.$$

Similarly, $\mathfrak{S}_{a_3,1} = 35$. Since $\mathfrak{S}_{a_1,a_3} = 45 > \mathfrak{S}_{a_3,a_1} = 35$, agent a_1 outweighs agent a_3, i.e. $(U;C) = (45, 0, 35 : \{a_1, a_3\}, a_2)$ is not in the Kernel. What about the $(U;CS) = (50, 30, 25; \{a_1, a_2, a_3\})$? The excesses and maximum surplus between agent a_2 and a_3 are:

$$\mathfrak{S}_{a_2,a_3} = max(\upsilon_{a_2} - U_{a_2}, \upsilon_{a_1,a_2} - U_{a_2}) = max(0 - 30, 90 - 30) = 60.$$

$$\mathfrak{S}_{a_3,a_2} = max(\upsilon_{a_3} - U_{a_3}, \upsilon a_1, a_3 - U_{a_3}) = max(0 - 25, 80 - 25) = 55.$$

Agent 2 outweighs agent 3, i.e. (50,30,25; {1,2,3}) is not in Kernel. What about the $(U; C) = (45, 35, 25; \{1, 2, 3\})$? The excesses and maximum surpluses are:

$\mathfrak{S}_{a_1,a_2} = max(0 - 45, 80 - 70) = 10 = max(0 - 35, 70 - 60) = \mathfrak{S}_{a_2,a_1}$,

i.e. player a_1 and a_2 are in equilibrium.

$\mathfrak{S}_{a_1,a_3} = max(0 - 45, 90 - 80) = 10 = max(0 - 25, 70 - 60) = \mathfrak{S}_{a_3,a_1}$,

i.e. player a_1 and a_3 are in equilibrium.

$\mathfrak{S}_{a_2,a_3} = max(0 - 35, 90 - 80) = 10 = max(0 - 25, 80 - 70) = \mathfrak{S}_{a_3,a_2}$,

i.e. player a_2 and a_3 are in equilibrium.

Hence, the $(U; C) = (45, 35, 25; \{a_1, a_2, a_3\})$ is in the Kernel.

3.5.2 The Core

Von Neumann and Morgenstern [17] consider that searching for stable states in a cooperative game is actually searching for payoff vectors that satisfy all agents. Hence, there is no incentive for any agent to deviate. They propose the idea of *individual rationality* that states that an agent in a coalition will never accept any payoff less than what it could receive from its singleton coalition, $U_i \geq \upsilon_i$ for all i. This individual rationality is virtually part of every stable state otherwise there will be at least one agent who deviates in order to satisfy this condition. Von Neumann et al. [17] also propose that agents should form coalitions such that the sum of coalition values is maximal. This is referred to as *group rationality* [17]. Since the environment they study is superadditive, υ_M is the largest coalition value. It can be claimed that agents should refuse any payoff configuration such that $\sum U_{i,i \in M} < \upsilon_N$. Von Neumann et al. [17] define group rationality as "the sum of every agent's payoff in the grand coalition is equal to the the grand coalition's value",$\sum U_{i,i \in M} = \upsilon_M$. The implication of this assumption into the general case is that agents should try to maximize the system's utility. A payoff vector that satisfies individual and group rationality is called an *imputation*.

Based on these rationalities, Gillies [18] defines the last level of rationality, i.e., *coalitional rationality* which requires that the sum of payoffs of agents in any coalition is not less than the coalition value, $\sum U_T \geq \upsilon_T$ for every $T \subseteq M$. There is no incentive for any agent to leave its coalition. Hence, it brings stability to the system. Gillies [18] names the set of payoff vectors that satisfy all three levels of rationality as the *Core*. Of all existing solution concepts in cooperative game theory, the core is simply the most beneficial concept that brings the most wealth to individual agents, coalitions, and the system as a whole. However, it is the hardest to satisfy as we shall cover in detail later.

Let us consider a simple game of 3 agents in a superadditive environment, explained in [48]. The game is defined by the characteristic function below:

$$\upsilon_{a_1} = \upsilon_{a_2} = \upsilon_{a_3} = 0$$

$$\upsilon_{a_1,a_2} = 0.25, \upsilon_{a_1,a_3} = 0.5, \upsilon_{a_2,a_3} = 0.75$$

$$\upsilon_{a_1,a_2,a_3} = 1$$

A payoff vector $(U_{a_1}, U_{a_2}, U_{a_3})$ is in the core of the game if

$$U_{a_1} + U_{a_2} \geq \upsilon_{a_1,a_2} = 0.25$$

$$U_{a_1} + U_{a_3} \geq \upsilon_{a_1,a_3} = 0.5$$

$$U_{a_2} + U_{a_3} \geq \upsilon_{a_2,a_3} = 0.75$$

Payoff vector (0.25, 0.5, 0.25), for example, meets all three conditions, so it is in the core.

Let us consider another example, the House Selling game discussed in [17], [48] which was analyzed by Von Neumann et al. [17]. Agent a_1 has a house which it values at \$100,000 and wants to sell it. Agents a_2 and a_3 are potential buyers, who each has \$200,000 in cash and values the house at \$200,000. The coalition value, in this case, is actually the difference between the amount that buyers and seller value the house. Because any singleton coalition and a coalition of buyers cannot make any deal, hence their values are 0. A coalition of agent a_1 and one of the buyers is \$100,000. The grand coalition also (theoretically) has the value \$100,000 (although the house can not be divided). Hence, the characteristic function is shown below:

$$\upsilon_{a_1} = \upsilon_{a_2} = \upsilon_{a_3} = 0$$

$$\upsilon_{a_1,a_2} = \upsilon_{a_1,a_3} = 100,000, \upsilon_{a_2,a_3} = 0$$

$$\upsilon_{a_1,a_2,a_3} = 100,000$$

The core of this game is the set of payoff vectors (U_1, U_2, U_3) with $U_1 + U_2 \geq 100,000$, $U_1 + U_3 \geq 100,000$ and $U_2 + U_3 \geq 0$. The only payoff vector which satisfies these conditions is (10,000,000). It implies that agent $a_1$sells the house to either agent a_1 or a_2 with the maximum possible price of \$200,000. The only agent who gets all the benefit of this economic cooperation is agent a_1. The negotiation that may lead to this conclusion could be agent a_2 offers \$150,000to agent a_1. Agent a_3 then raises the bid by offering agent a_1 \$175,000, and so on. As long as the offer is below \$200,000, there is a chance both agents can raise the bids.

Now, let us consider the Sandal game. Since the largest amount possible is \$600, the proposed payoff vector (100, 50, 100, 150, 0) is definitely not in the core because it is not group rational. Payoff Vector (120, 120, 120, 120, 120) is in the core because it satisfies individual, group and coalitional rationality. No agent can deviate and be better off.

3.6 Shapley Value

There are several solution concepts in coalition formaiton, Shapley value[39]. Here, we briefly review Shapley value as it is the main issue of the paper.

Shapley Value is calculated based on each agent's contribution to the value of the coalition. Starting from the beginning, we consider that a coalition S_{i+} is formed by a new agent a_i joining an existing coalition S. The increased value of the new coalition is the contribution of the joining agent to the coalition. The number of ways or orders of which agents join the coalition is actually the number of permutations of members of that coalition. The payoff for each agent is average contribution of the agent over all permutation.

Given a characteristic function game, we can summarize that there are two steps in computing Shapley value of a coalition.

1. For the given coalition, compute all the permutations of coalition members.

2. For each permutation, compute the contribution of each agent, starting from the first agent on the left, to the last member on the right

3. For each agent, sum the contribution of each agent over all permutation and divide it with the number of permutations for the average contribution, which is the agent's payoff. See Algorithm 1.

3.6.1 Shapley Value Algorithm

Algorithm 1 Shapley Value Algorithm

procedure
    Set a = noa
    for each $p \in P$ **do**
        Set a_j = null
        for each $a_j \in p$ **do**
            Set $S(a_j) = S(a_k) \bigcup a_j$
            $C^P_{a_k} = v(S_{a_k}) - v(S_{a_j})$
            $S(a_j) = S(a_k)$
        end for
        for each $a_j \in a$ **do**
            $C(a_j, p) = \sum_{i=0}^{P} C^P_{a_k}$
            $S(a_j) = S(a_k)$
        end for
        $Arg = 1/a \; C_{(a_j,p)}$
    end for

For example:

consider characteristic function of this game following :

$$v(a_1) = 2, v(a_2) = 4, v(a_3) = 3,$$

$$v(a_1, a_2) = 5, v(a_1, a_3) = 6, v(a_1, a_3) = 8,$$

$$v(a_1, a_2, a_3) = 10,$$

In step 1),Computing all the permutation from number of agents in coalition that contribute payoffs by n! so from the example there are 3 agents = 3!, this is 6 permutations:

$$\{a_1, a_2, a_3\}, \{a_1, a_3, a_2\}, \{a_2, a_1, a_3\},$$

$$\{a_2, a_3, a_1\}, \{a_3, a_1, a_2\} \text{ and} \{a_3, a_2, a_1\}.$$

In step 2), we compute the contribution of each agent. At the beginning, there is no member in the coalition. The value $\upsilon(S))$ is $\emptyset$. Then agent a_1 joins the coalition. Its contribution is

$$\delta(\upsilon) = \upsilon(S_{i^+}) - \upsilon(S) = 2 - \emptyset = 0.$$

The next coalition member is then a_2, its contribution is

$$\upsilon_{(S_{a_1,a_2})} - \upsilon_{(S_{a_1})} = 5 - 2 = 3.$$

Hence, the contribution of agent a_3 is

$$\upsilon_{(S_{a_1,a_2,a_3})} - \upsilon_{(S_{a_1,a_2})} = 10 - 5 = 5.$$

We repeat this calculation for each agent in the remaining permutations. The contributions of each agent are shown in the table 3.2.

In step 3), we sum the contributions of each agent,
Contribution of agent a_1 :

2 + 2 + 1 + 2 + 3 + 2 = 12

Contribution of agent a_2 :

3 + 4 + 4 + 4 + 4 + 5 = 24

Contribution of agent a_3 :

5 + 4 + 5 + 4 + 3 + 3 = 24.

In step 4), divide the contribution with 6 permutations for the payoff of each agent, i.e.

Payoff of agent $a_1 = 2$

Payoff of agent $a_2 = 4$

Payoff of agent $a_3 = 4.$

According to the results in the table 3.2.

Table. 3.2. For example Contribution By Shapley value

Order	Contribution of Agents		
	a_1	a_2	a_3
$\{a_1, a_2, a_3\}$	2	3	5
$\{a_1, a_3, a_2\}$	2	4	4
$\{a_2, a_1, a_3\}$	1	4	5
$\{a_2, a_3, a_1\}$	2	4	4
$\{a_3, a_1, a_2\}$	3	4	3
$\{a_3, a_2, a_1\}$	2	5	3
Sum of Contribution	12	24	24
Average Contribution	2	4	4

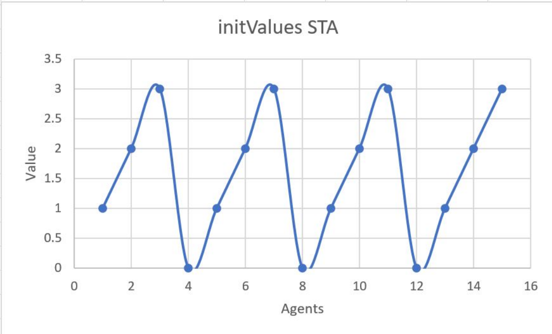
3.7 Experiment Setting

We assume superadditive environment. It is important that this characteristic must be preserved. This must be done carefully through every subset when a new coalition is formed so that $v_{(S \cup T)} \geq v_{(S)} + v_{(T)}$ is consistent. The Cup and Cap as shown in Figure 3.1 (d) and (e) various shapes of distribution can be done at cardinality: 1) Consequent coalitions may not have the shapes, 2) Non-superadditive Data distributions is preserved at all cardinalities.

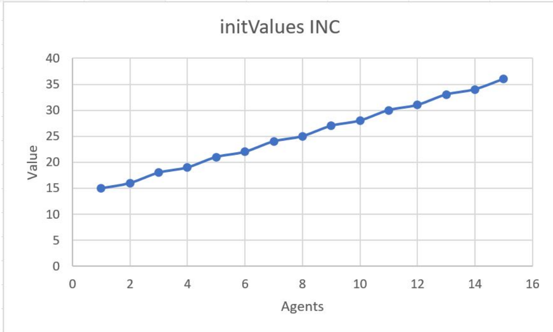
Table. 3.3. Setting value of agent

Agent	0-1	0-2	0-3	0-4	0-5	0-6	0-7	0-8
%	50%	30%	25%	20%	16%	14%	12%	11%
Agent	0-9	0-10	0-11	0-12	0-13	0-14	0-15	
%	10%	6%	5%	4%	3%	2%	1%	

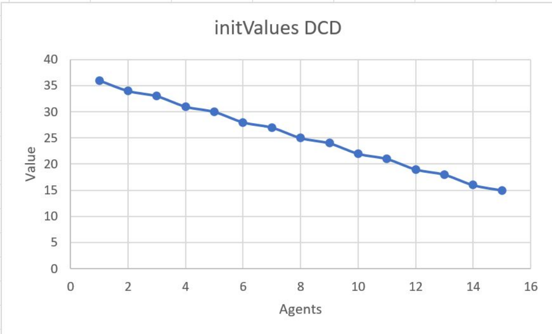
For each of these setting, the coalition value will be initialize as per the tread of singleton agents value. Then the new coalition will be formed by adding each agents to each of the available coalition and the new coalition value will be generated by a random in uniform distribution function to increase the value by to 3. This process is repeat for 14 time, i.e.,form conditionality 1 to 15. Then the whole process is repeated for 100 times. The final payoff for agents are teen averaged.



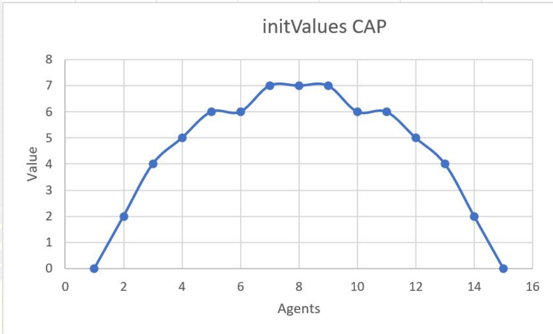
(a) STA.



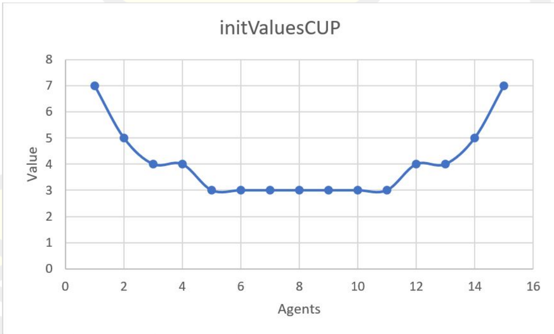
(b) INC.



(c) DCD.



(d) CAP.



(e) CUP.

Figure. 3.1. Initiative value.

Figure 3.1 show 5 Data Distribution types, including. 1) STA, 2) IND, 3) DCD, 4) Cup and 5) Cap,by consider the following example of CS that have coalition values listed in $|S|$, that 2 cardinalities including $|S| = 4, 1$ and $|S| = 3, 2$. The details are as follows data distribution used to define characteristic function divided into 5 data distribution types including

1) STA: This is uniform distribution that the value of CS is always the same, which value in 2 cardinalities as shown in table 3.4.

Table. 3.4. For example Increase value of coalition in superadditive

$\lvert S \rvert$ = **{4}, {1}**			$\lvert S \rvert$ = **{3}, {2}**	
CS of agents	**Value**		**CS of agents**	**Value**
$\{1,2,3,4\},\{5\}$	15		$\{3,4,5\},\{1,2\}$	16
$\{1,2,3,5\},\{4\}$	16		$\{2,4,5\},\{1,3\}$	17
$\{1,2,4,5\},\{3\}$	22		$\{2,3,5\},\{1,4\}$	18
$\{1,3,4,5\},\{2\}$	23		$\{2,3,4\},\{1,5\}$	19
$\{2,3,4,5\},\{1\}$	24		$\{1,4,5\},\{2,3\}$	20
			$\{1,3,5\},\{2,4\}$	21
			$\{1,3,4\},\{2,5\}$	22
			$\{1,2,5\},\{3,4\}$	23
			$\{1,2,4\},\{3,5\}$	24
			$\{1,2,3\},\{4,5\}$	25

2) INC: The maximal coalition values increase by the cardinalities. (Note that although this is similar to superadditive but it is not quite the same.) which value in 2 cardinalities. as shown in table 3.5

Table. 3.5. For example Uniform value of coalition in superadditive

$\|S\|$ = **{4}, {1}**			$\|S\|$ = **{3}, {2}**	
CS of agents	**Value**		**CS of agents**	**Value**
$\{1,2,3,4\},\{5\}$	7		$\{3,4,5\},\{1,2\}$	10
$\{1,2,3,5\},\{4\}$	7		$\{2,4,5\},\{1,3\}$	10
$\{1,2,4,5\},\{3\}$	7		$\{2,3,5\},\{1,4\}$	10
$\{1,3,4,5\},\{2\}$	7		$\{2,3,4\},\{1,5\}$	10
$\{2,3,4,5\},\{1\}$	7		$\{1,4,5\},\{2,3\}$	10
			$\{1,3,5\},\{2,4\}$	10
			$\{1,3,4\},\{2,5\}$	10
			$\{1,2,5\},\{3,4\}$	10
			$\{1,2,4\},\{3,5\}$	10
			$\{1,2,3\},\{4,5\}$	10

3) DCD: The maximal coalition values decrease by the cardinalities. Again, it is similar but is not subadditive, whose value in 2 cardinalities as shown in table 3.6

Table. 3.6. For example DCD value of coalition in superadditive

$\|S\|$ = **{4}, {1}**			$\|S\|$ = **{3}, {2}**	
CS of agents	**Value**		**CS of agents**	**Value**
$\{1,2,3,4\},\{5\}$	20		$\{3,4,5\},\{1,2\}$	25
$\{1,2,3,5\},\{4\}$	19		$\{2,4,5\},\{1,3\}$	24
$\{1,2,4,5\},\{3\}$	18		$\{2,3,5\},\{1,4\}$	23
$\{1,3,4,5\},\{2\}$	17		$\{2,3,4\},\{1,5\}$	22
$\{2,3,4,5\},\{1\}$	16		$\{1,4,5\},\{2,3\}$	21
			$\{1,3,5\},\{2,4\}$	20
			$\{1,3,4\},\{2,5\}$	19
			$\{1,2,5\},\{3,4\}$	18
			$\{1,2,4\},\{3,5\}$	17
			$\{1,2,3\},\{4,5\}$	18

4) Cap: This is normal distribution which value in 2 cardinalities as shown in table 3.7

Table. 3.7. For example Cap value of coalition in superadditive

$\lvert S \rvert$ **= {4}, {1}**			$\lvert S \rvert$ **= {3}, {2}**	
CS of agents	**Value**		**CS of agents**	**Value**
$\{1,2,3,4\},\{5\}$	17		$\{3,4,5\},\{1,2\}$	16
$\{1,2,3,5\},\{4\}$	26		$\{2,4,5\},\{1,3\}$	18
$\{1,2,4,5\},\{3\}$	30		$\{2,3,5\},\{1,4\}$	21
$\{1,3,4,5\},\{2\}$	27		$\{2,3,4\},\{1,5\}$	25
$\{2,3,4,5\},\{1\}$	16		$\{1,4,5\},\{2,3\}$	30
			$\{1,3,5\},\{2,4\}$	27
			$\{1,3,4\},\{2,5\}$	23
			$\{1,2,5\},\{3,4\}$	18
			$\{1,2,4\},\{3,5\}$	15
			$\{1,2,3\},\{4,5\}$	14

5) Cup: The maximal coalition values on cardinalities 1 and n are high and decrease towards the medium cardinalities, whose value in 2 cardinalities as shown in table. 3.8

Table. 3.8. For example Cup value of coalition in superadditive

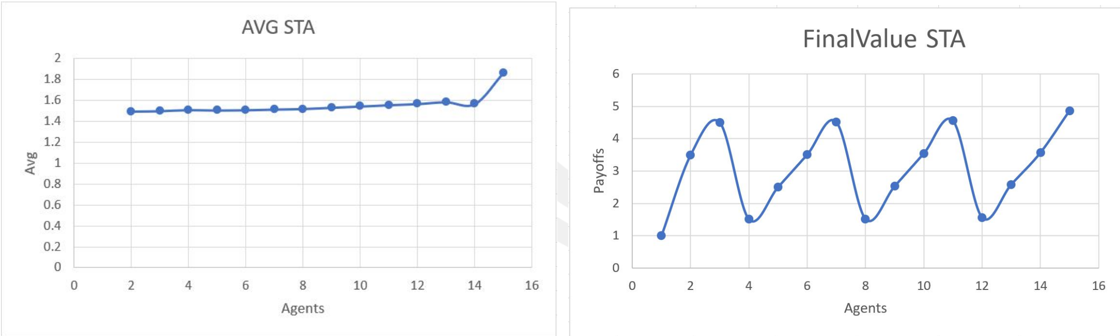
$\lvert S \rvert$ = **{4}, {1}**		$\lvert S \rvert$ = **{3}, {2}**	
CS of agents	**Value**	**CS of agents**	**Value**
$\{1,2,3,4\},\{5\}$	32	$\{3,4,5\},\{1,2\}$	34
$\{1,2,3,5\},\{4\}$	21	$\{2,4,5\},\{1,3\}$	23
$\{1,2,4,5\},\{3\}$	16	$\{2,3,5\},\{1,4\}$	20
$\{1,3,4,5\},\{2\}$	19	$\{2,3,4\},\{1,5\}$	18
$\{2,3,4,5\},\{1\}$	30	$\{1,4,5\},\{2,3\}$	17
		$\{1,3,5\},\{2,4\}$	19
		$\{1,3,4\},\{2,5\}$	22
		$\{1,2,5\},\{3,4\}$	24
		$\{1,2,4\},\{3,5\}$	31
		$\{1,2,3\},\{4,5\}$	32

3.8 Results

In this section, we assign different patterns of coalition values and observe the the average and final payoffs of agents for different coalition sizes. There are five distribution patterns to consider, including STA, INC, DCD, CAP and CUP.

3.8.1 STA Results

As shown in Figure 3.2 (a), the average payoffs tend to increase very slowly when the size of coalition increases. However, the final payoffs for agents have similar patterns to the coalition values, as shown in Figure 3.2 (b).

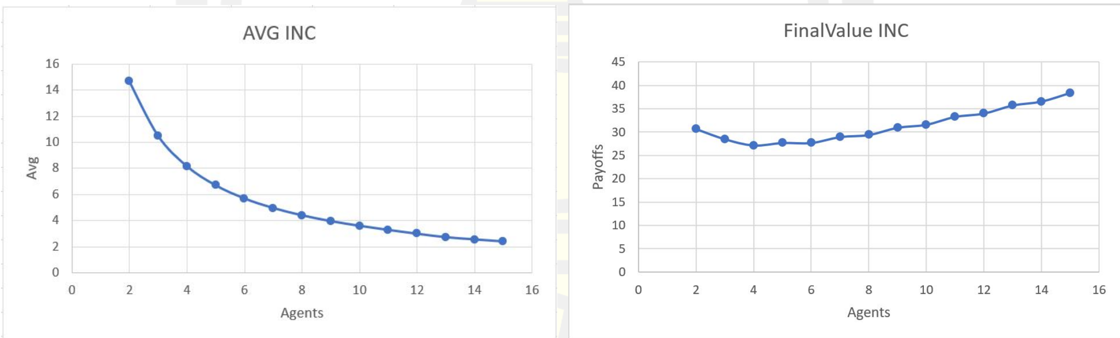


(a) Average payoffs of STA **(b)** Final payoffs of STA

Figure. 3.2. STA Result Average payoffs (a) and final payoffs (b).

3.8.2 INC Results

As shown in Figure 3.3 (a), the average payoffs tend to decrease sharply for small coalitions. For medium an large coalitions, their average payoffs decrease slowly. However, the final payoffs for agents have similar patterns to the coalition values, as shown in Figure 3.3 (b).

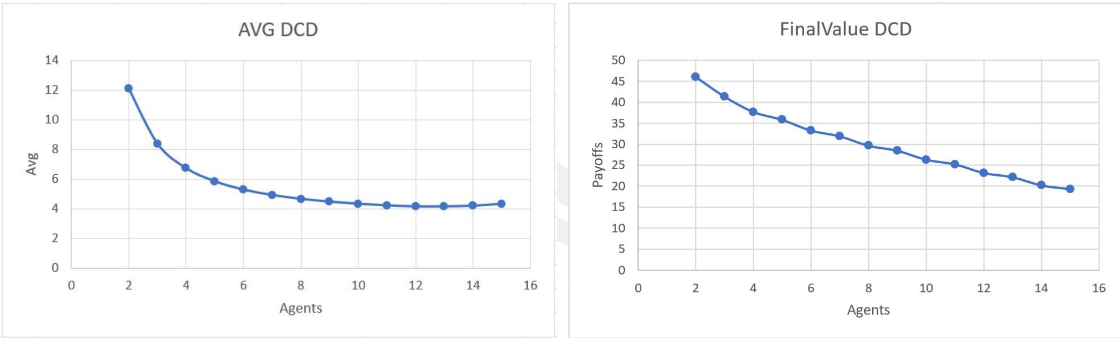


(a) Average payoffs of INC **(b)** Final payoffs of INC

Figure. 3.3. INC Result Average payoffs (a) and final payoffs (b).

3.8.3 DCD Results

As shown in Figure 3.4 (a), the average payoffs tend to decrease sharply for small coalitions. For medium an large coalitions, their average payoffs are quite stable. However, the final payoffs for agents have similar patterns to the coalition values, as shown in Figure 3.4 (b).

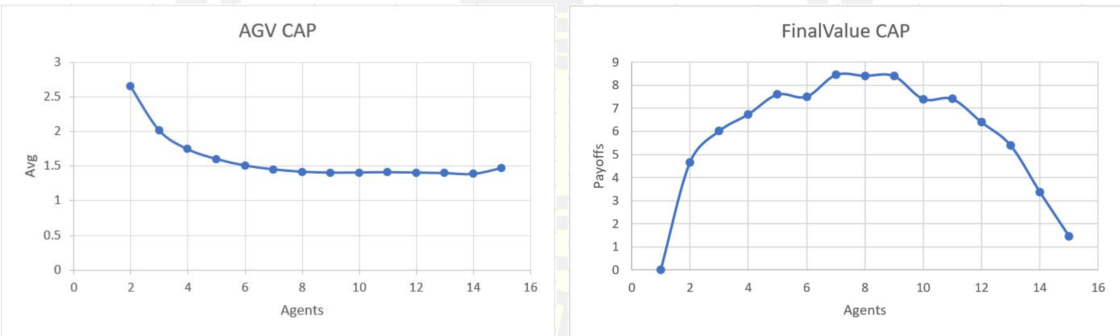


(a) Average payoffs of DCD **(b)** Final payoffs of DCD

Figure. 3.4. DDC Result Average payoffs (a) and final payoffs (b).

3.8.4 CAP Results

As shown in Figure 3.5 (a), the average payoffs tend to decrease sharply for small coalitions. For medium an large coalitions, their average payoffs are quite stable. However, the final payoffs for agents have similar patterns to the coalition values, as shown in Figure 3.5 (b).



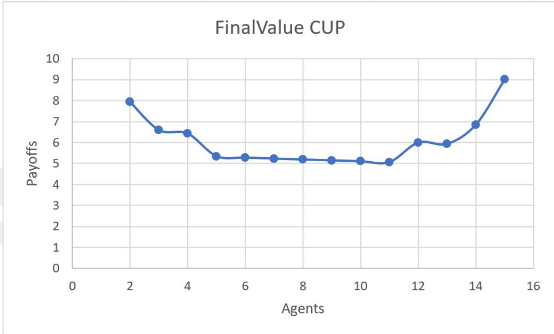
(a) Average payoffs of CAP **(b)** Final payoffs of CAP

Figure. 3.5. CAP Result Average payoffs (a) and final payoffs (b).

3.8.5 CUP Results

As shown in Figure 3.6 (a), the average payoffs tend to decrease when the size of the coalition increases. However, the final payoffs for agents have similar patterns to the coalition values, as shown in Figure 3.6 (b).

(a) Average payoffs of CUP



(b) Final payoffs of CUP

Figure. 3.6. CUP Result Average payoffs (a) and final payoffs (b).

3.9 conclusion

In this chapter, we explore the final payoffs of agents whether they are affected by any pattern of coalition values under Shapley value. We have five coalition value distribution patterns, namely STA, INC, DEC, CAP and CUP. As we can see, the final agents payoffs are still in the same trends as their original values. However, the average payoffs of agents in coalition of different sizes are affected by these patterns.

CHAPTER 4

FAIR PAYOFF IN LINEAR PRODUCTION GAME

In this chapter the we turn our attention to the classic linear production game and a couple of its variants. In contrast to characteristic function game in which the coalition value of each coalition is known a prior, agents are given resources and valeted information than compete coalition values for making decision. This is more realistic in real world settings. We want to find out relationship of agents resources, their values, agents decisions and their payoffs.

In the following sections, we will firstly review the original works that we based our research upon, i.e. game [40, 59]. For the sake of completeness, we briefly review these works in section 4.1, 4.2, and 4.3, as they were presented. These works generally do not take into account the computation for payoffs. However, they propose two different settings. Then we will present our own work in section 4.4, 4.5 examine agent payoffs given different resources distribution and conclude.

4.1 Generic Case

4.1.1 Linear Production Game

Here, we present the setting of linear production game [40] in non-superadditive environments, which is the foundation of our work.

Let $A = \{a_1, a_2, ..., a_{\mathbf{m}}\}$ be a set of **m** agents, whose goals are to maximize the their individual profit. Let $R = \{r_1, r_2 ..., r_{\mathbf{n}}\}$ be a set of **n** resources. Let $G = \{g_1, g_2, ..., g_{\mathbf{o}}\}$ be a set of **o** goods. Resources themselves are not valuable but they can be used to produce goods. The *linear technology matrix* [40] $L = [\alpha_{i,j}]_{\mathbf{n}\times\mathbf{o}}$, where $\alpha_{i,j} \in \mathbb{Z}^+$, $1 \le i \le \mathbf{n}$ and $1 \le j \le \mathbf{o}$, specifies the units of each resource $r_i \in R$ required to produce a unit of the good $g_j \in G$. The price of each unit of goods produced is specified by the vector $P = [p_j]_{1\times\mathbf{o}}$. Each agent $a_k \in A$ where $1 \le k \le \mathbf{m}$, is given a resource bundle. The Available Resource matrix $B = [\beta_{i,k}]_{\mathbf{n}\times\mathbf{m}}$ specifies the number of resource r_i possessed by agent a_k. In this setting, some agents would have the incentive to cooperate, e.g., if they cannot produce a certain good using only the resources at their disposal. Hence agents have to cooperate, i.e. form coalitions, in order to create value from their resources. Let $S \subseteq A$ be a coalition. It will have a total of

$$b_i = \sum_{1 \leq k \leq \mathbf{m}} \beta_{i,k} \tag{4.1}$$

of the i^{th} resource. The members of coalition S can use all these resources to produce any vector $x = \langle x_1, x_2, \ldots, x_{\mathbf{o}} \rangle$ of goods constraints:

$$\begin{array}{rcl} \alpha_{1,1}x_1 + \alpha_{1,2}x_2 + \ldots + \alpha_{\mathbf{o},1}x_{\mathbf{o}} & \leq & b_1, \\ \alpha_{2,1}x_1 + \alpha_{2,2}x_2 + \ldots + \alpha_{\mathbf{o},2}x_{\mathbf{o}} & \leq & b_2, \\ \vdots & \vdots & \vdots, \\ \alpha_{\mathbf{m},1}x_1 + \alpha_{\mathbf{m},2}x_2 + \ldots + \alpha_{\mathbf{o},\mathbf{m}}x_{\mathbf{o}} & \leq & b_{\mathbf{m}} \end{array} \tag{4.2}$$

and

$$x_1, x_2, \ldots, x_{\mathbf{o}} \geq 0. \tag{4.3}$$

We assume that agents have to pool their resources together at a coalition member's location to produce these goods. This game can be summarized by Table 4.1.

Table. 4.1. Formulation Linear Production Game

Agent				Total Amount of Resource	L				**Resource**
a_1	a_2	...	$a_{\mathbf{m}}$		g_1	g_2	...	$g_{\mathbf{o}}$	
$\beta_{1,1}$	$\beta_{1,2}$	...	$\beta_{1,\mathbf{m}}$	b_1	$\alpha_{1,1}$	$\alpha_{1,2}$	...	$\alpha_{1,\mathbf{o}}$	r_1
$\beta_{2,1}$	$\beta_{2,2}$	...	$\beta_{2,\mathbf{m}}$	b_2	$\alpha_{2,1}$	$\alpha_{2,2}$	...	$\alpha_{2,\mathbf{o}}$	r_2
		$\vdots$		$\vdots$			$\vdots$		$\vdots$
$\beta_{\mathbf{n},1}$	$\beta_{\mathbf{n},2}$	...	$\beta_{\mathbf{n},\mathbf{m}}$	$b_{\mathbf{n}}$	$\alpha_{\mathbf{n},1}$	$\alpha_{\mathbf{n},2}$	...	$\alpha_{\mathbf{n},\mathbf{o}}$	$r_{\mathbf{n}}$
				Price	p_1	p_2	...	$p_{\mathbf{o}}$	

Thus agents' cooperation incurs some costs, e.g., transportation cost, etc. The cooperation cost among agents is specified by the matrix $C = [c_{kl}]_{m \times m}$, which assigns a cooperation cost between each pair (a_k, a_l) of agents such that

$$c_{kl} \in \begin{cases} \mathbb{Z}^+ & \text{if } k \neq l \\ \{0\} & \text{if } k = l \end{cases} \tag{4.4}$$

We assume that all of the resources of agents are pooled at one location, which can be the location of any agent in the coalition. A singleton coalition yields cooperation cost of 0. For a coalition of size two, $S = \{a_1, a_2\}$, pooling coalition resources at any of the two sites yield the same cost for the coalition (i.e. the cooperation cost matrix is symmetric). The total cost for cooperation incurred by a coalition will be taken to be the sum of the pairwise cooperation costs between the agent at whose location coalition resources are pooled, and the other members of coalition. For a coalition of size three or larger, there is at least one agent, a_k, such that

$$\sum_{k'=1}^{m} c_{kk'} \leq \sum_{l'=1}^{m} c_{ll'} \tag{4.5}$$

for all $a_l \in S$. We shall call a coalition member a_k who yields the minimal cooperation cost for the coalition a *coalition center*.

Agents in the coalition S have to find a vector x to maximize the revenue accruing to a coalition. Let

$$P_S = \sum_{l=1}^{o} p_l x_l. \tag{4.6}$$

be the maximal revenue the coalition can generate. Let

$$C_S = \sum_{l \in S} c_{kl}. \tag{4.7}$$

be the minimal cooperation cost for the coalition (obtained by selecting the optimal coalition center). Obviously, the ultimate objective of agents in the coalition is to maximize profit, i.e., the coalition value υ_S, where

$$\upsilon_S = P_S - C_S. \tag{4.8}$$

The linear inequalities referred to above, together with this objective function constitutes a linear programming problem. We shall call the solution, the vector

$\langle x_1, x_2, \ldots, x_o \rangle$ that represents the optimal quantities of goods $g_1, g_2, \ldots, g_o$ *optimal product mix.*

4.2 Case Study: Game 1

Based on the original linear production game [40], Owen further investigate for an outcome of a game by considering a production game [40] with two resources, and three types of players, with initial resource bundles

$$b^1 = (6, 1), \quad b^2 = (4 - \sqrt{2}, 4), \quad b^3 = (\sqrt{2}, 5)$$

Owen assumes that is only one product, a unit of which requires one unit of each of the resources, and can be sold for \$2. then

$$b(N) = (10, 10)$$

and the dual problem takes the form

$$v(N) = min 10y_1 + 10y_2,$$

$$\text{subject to } y_1 + y_2 \geq 2, \quad y_1, y_2 \geq 0.$$

It is found that the equilibrium here is not unique; in fact, any vector

$$y_1 + y_2 = 2, \quad y_1 \geq 0, \quad y_2 \geq 0$$

will solve the linear program. In particular $y_1 = y_2 = 1$ will solve the program, giving the payoffs $u_1 = 7, u_2 = 8 - \sqrt{2}, u_3 = 5 + \sqrt{2}$ to the three types.

Owen then consider, the payoff

$$u(\in) = (7+ \in_1, 8 - \sqrt{2}+ \in_2, 5 + \sqrt{2}+ \in_3),$$

where the $\in_i$ are small numbers, satisfying $\in_1 + \in_2 + \in_3 = 0$. Owen shows that for any r, $u(\in)$ will belong to the core for all sufficiently small $\in$.

In fact, suppose S has Z_i players of type $i(i = 1, 2, 3)$, where $z_i \geq r$. Then

$$b_1(S) = 6z_1 + (4 - \sqrt{2})z_2 + \sqrt{2}z_3,$$

$$b_2(S) = z_1 + 4z_2 + 5z_3,$$

and so, since each unit of the product sells for \$2.00

$$v(S) = min \begin{cases} 12z_1 + (8 - 2\sqrt{2})z_2 + \sqrt{2}z_3, \\ 2z_1 + 8z_2 + 10z_3. \end{cases}$$

Since $v(S)$ is the smaller of these two numbers, it cannot be more than half their sum, and so there exists the inequality

$$v(S) \leqslant 7z_1 + (8 - \sqrt{2})z_2 + (5 + \sqrt{2})z_3 \tag{4.9}$$

with equality holding *if and only if*

$$12z_1 + (8 - 2\sqrt{2})z_2 + 2\sqrt{2}z_3 = 2z_1 + 8z_2 + 10z_3$$

or

$$10z_1 - 2\sqrt{2}z_2 + (2\sqrt{2} - 10)z_3 = 0$$

Since the z_i are integers, this can only happen if $z_2 = z_3$ (as this will cause the irrational terms to vanish). Letting $z_2 = z_3$, it holds that

$$10z_1 - 10z_3 = 0$$

and so $z_1 = z_3$. Thus, *in*(4.9), Owen states that *equality will hold if and only if*

$$z_1 = z_2 = z_3.$$

In this case, of course, it holds that

$$v(S) = 20z_1$$

and, letting $u(S)$ be the total payoff to the coalition S,

$$u(S) = (7+ \in_1)z_1 + (8 - \sqrt{2}+ \in_2)z_2 + (5 + \sqrt{2}+ \in_3)z_3$$

but, with $z_1 = z_2 = z_3$, this takes form

$$u(s) = 20z_1 + (\in_1 + \in_2 + \in_3)z_1$$

However, the assumption $\in_1 + \in_2 + \in_3 = 0$ is made, and so $u(S) = 20z_1$. Hence

$$v(S) \leqslant u(s).$$

Suppose, in the other hand, equality dose *not* hold in $x_k, z_i \geqslant 0$,. Then it holds that

$$v(S) = 7z_1 + (8 - \sqrt{2})z_2 + (5 + \sqrt{2})z_3 - \delta,$$

where $\delta > 0$. On the other hand, it holds that

$$u(S) = (7+ \in_1)z_1 + (8 - \sqrt{2}+ \in_2)z_2 + (5 + \sqrt{2}+ \in_3)z_3$$

or

$$u(S) = v(S) + \delta + \in_1 z_1 + \in_2 z_2 + \in_3 z_3.$$

Choose, now, $\in_i$ to satisfy

$$| \in_i | < \frac{\delta}{3r} \tag{4.10}$$

Since $z_i \leqslant r$, it holds that

$$| \in_1 z_1 + \in_2 z_2 + \in_3 z_3| < \frac{\delta}{3r}(z_1 + z_2 + z_3) \leqslant \delta$$

and so

$$u(S) \geqslant v(S).$$

Owen states that no matter how large r is, the vector $u(\in)$ will be in the core tor all $\in$ satisfying (4.16), with $\in_1 + \in_2 + \in_3 = 0$. But this gives us a two-dimensional set of core imputation. Since the equilibrium payoffs form only a one-dimensional set, it is clear that the mapping $u_i = b_1^i y_1^* + b_2^i y_2^* + ... + b_m^i y_m^*$ cannot give rise to a two-dimensional set, and owen conclude that. for any r, the core will contain non-equilibrium payoffs. In other words, convergence here requires an infinite number of steps.

Here, we have different settings from that of owen. In our environment, we will explore the outcome of game where resources are provided and will be computed for prices and payoffs.

4.3 Case Study: Game 2

The original a linear production game [40] was extended in many ways. One of these extended game is Tijis's [59], which is defined by

$$\begin{array}{c} \\ P_1 \\ P_2 \end{array} \begin{array}{c} \begin{array}{cc} G_1 & G_2 \end{array} \\ \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \end{array} \quad \begin{array}{c} \\ \begin{pmatrix} 5 \\ 7 \end{pmatrix} \end{array}$$

$$\begin{aligned} b^1 &= (5,8) \\ b^2 &= (5,2) \\ b^3 &= (0,2) \end{aligned} \tag{4.11}$$

As given in this situation (4.11), where 2 resource G_1 and G_2 are involved and 2 products P_1 and P_2, with prices per unit 5 and 7. Suppose that 10 units of G_1 and 12 units of G_2 are available: player 1, 2 and 3 own, respectively, resource bundles (5, 8), (5, 2) and (0, 2). This situation corresponds to a 3-person linear production game $< N, v >$ with

$$v(S) := max\{5x_1 + 7x_2 | x_1 \geqslant 0, x_2 \geqslant 0, x_1 + 2x_2 \leqslant b_1(S), 2x_1 + x_2 \leqslant b_2(S)\} \tag{4.12}$$

where $b_k(S) = (\sum_{i \in S} b^i)_k$ for $k \in \{1, 2\}$ denotes the total amount of resource G_k owned by coalition S.

Tijis's [59] states that i v, is derived from the duality theorem of linear programming theory. The reason is that for all dual programs corresponding to the $2^n - 1 = 7$

coalitions the feasible region is the same. It holds that

$$v(S) = min\{b_1(S)y_1 + b_2(S)y_2 | y_1 \geqslant 0, y_2 \geqslant 0, y_1 + 2y_2 \geqslant 5, 2y_1 + y_2 \geqslant 7\} \quad (4.13)$$

It can be seen that the feasible region of the dual problem has 3 extreme points, namely $\hat{y} = (3,1), \hat{y'} = (5,0), \hat{y''} = (0,7)$. For each of the 7 problems, the minimum is attained at one of these extreme points. It is also shown that the minimum is attained and gives also the characteristic function v.

Table. 4.2. An example resource

S	$b(S)$	minimum	$v(S)$
(1)	$(5,8)$	$\hat{y}$	23
(2)	$(5,2)$	$\hat{y''}$	14
(3)	$(0,2)$	$\hat{y'}$	0
$(1,2)$	$(10,10)$	$\hat{y}$	40
$(1,3)$	$(5,10)$	$\hat{y}, \hat{y'}$	25
$(2,3)$	$(5,4)$	$\hat{y}$	19
$(1,2,3)$	$(10,12)$	$\hat{y}$	42

As shown in Table 4.2 the grand coalition N's the value is equal to 42 and is attained at $\hat{y} = (3,1)$. Here, 3 can be interpreted as (shadow) price for G_1. According to these prices the bundle $b^1 = (5,8)$, owned by player 1, has value $5 \cdot 3 + 8 \cdot 1 = 23$. For players 2 and 3 the values of their bundles are 17 and 2 receptively. These values correspond to the importation (23, 17, 2) of $< N, v >$. Note that (23, 17, 2) is even a core element of $< N, v >$ and that to find this vector we only need to solve the dual linear program in Table 4.2 for $S = N$.

In the Owen model each player i owns a bundle

$$b^i = (b^i_i, b^i_2, ..., b^i_q)$$

of resources and $b(S) := \sum_{i \in S} b^i$ for each

$$S \in 2^N \backslash \{0\}.$$

Tijis's [59] proposes that in the Granot model one considers the q commodity games $< N, c_k >$ with $c_k(S) := b_k(S)$ and assumes that these games are balanced.

In the Curiel-Derks-Tijs model for each commodity G_k there are portions

$$\alpha_{k1}, \alpha_{k2}, ..., \alpha_{kt(k)}$$

respectively. One assumes that the control of these games are simple games with veto players. Hence, $b_k(S) = \sum_{r=1}^{t(k)} \alpha_{kr} w_{kr}(S)$ for each S.

Tijis propose that in the Owen [40] model

$$b_k(S) = \sum_{i \in S} b_k^i = \sum_{i=1}^{n} b_k^i \delta_i(S)$$

for all $k \in N$ and $S \in 2^n \backslash \{0\}$, where δ_i is the dictator game with dictator i. This implies that this model is a special case of the third model by taking for each $k \in \{1, ..., q\} : t(k) = n$, and for each $i \in N : \alpha_{ki} = b_k^i, w_{ki} = \delta_i$

4.4 Experiment Setting and Result

Based on these works [40, 59], we proceed with different resource distributions. There are 13 agents and 6 resource of data distribution patterns in Figure 4.1.

images/C1.jpg

(a) IND.

images/C4.jpg

(b) DCD.

images/C2.jpg

(c) IND VS DCD.

images/C5.jpg

(d) CAP.

images/C3.jpg

(e) CUP.

images/C6.jpg

(f) CAP VS CUP.

Figure. 4.1. Data Distributions.

In IND, agent has the lowest quantity of both R_1 and R_2. The numbers of both resources increase / decrease for other agents.

Case 1: Superadditive IND

Table. 4.3. Game 1 and 2: IND Setting

		Agents												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Resource	**R1**	2	4	6	8	10	12	14	16	18	20	22	24	26
	R2	1	3	5	7	9	11	13	15	17	19	21	23	25

Case 2: Superadditive DCD

Table. 4.4. Game 1 and 2: DCD Setting

		Agents												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Resource	**R1**	26	24	22	20	18	16	14	12	10	8	6	4	2
	R2	25	23	21	19	17	15	13	11	9	7	5	3	1

Case 3: Superadditive IND VS DCD.

In IND VS DCD, agent 1 has the highest number of R_1 but has the lowest number of R_2. The number of R_1 is increased for other agents while the number of R_2 is decreed.

Table. 4.5. Game 1 and 2: IND VS DCD Setting

		Agents												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Resource	**R1**	2	4	6	8	10	12	14	16	18	20	22	24	26
	R2	26	24	22	20	18	16	14	12	10	8	6	4	2

Case 4: Superadditive CAP

In CAP, agent1 has the lowest number of R_1 and R_2 the numbers of both resources rise and reach the peals for agent 1 then decrease and reach the lowest for agent 13.

Table. 4.6. Game 1 and 2: Cap Setting

		Agents												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Resource	**R1**	2	4	6	9	11	13	14	13	11	9	6	4	2
	R2	1	3	5	8	10	12	13	12	10	8	5	3	1

Case 5: Superadditive CUP

In CUP, the pattern of resource distribution for all agents are opposite to CAP, i.e., agent 1 and 13 have the highest number of both resources where as agent 8 has the lowest.

Table. 4.7. Game 1 and 2: Cup Setting

		Agents												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Resource	**R1**	13	11	9	6	4	2	1	2	4	6	9	11	13
	R2	12	10	8	5	3	1	0	1	3	5	8	10	12

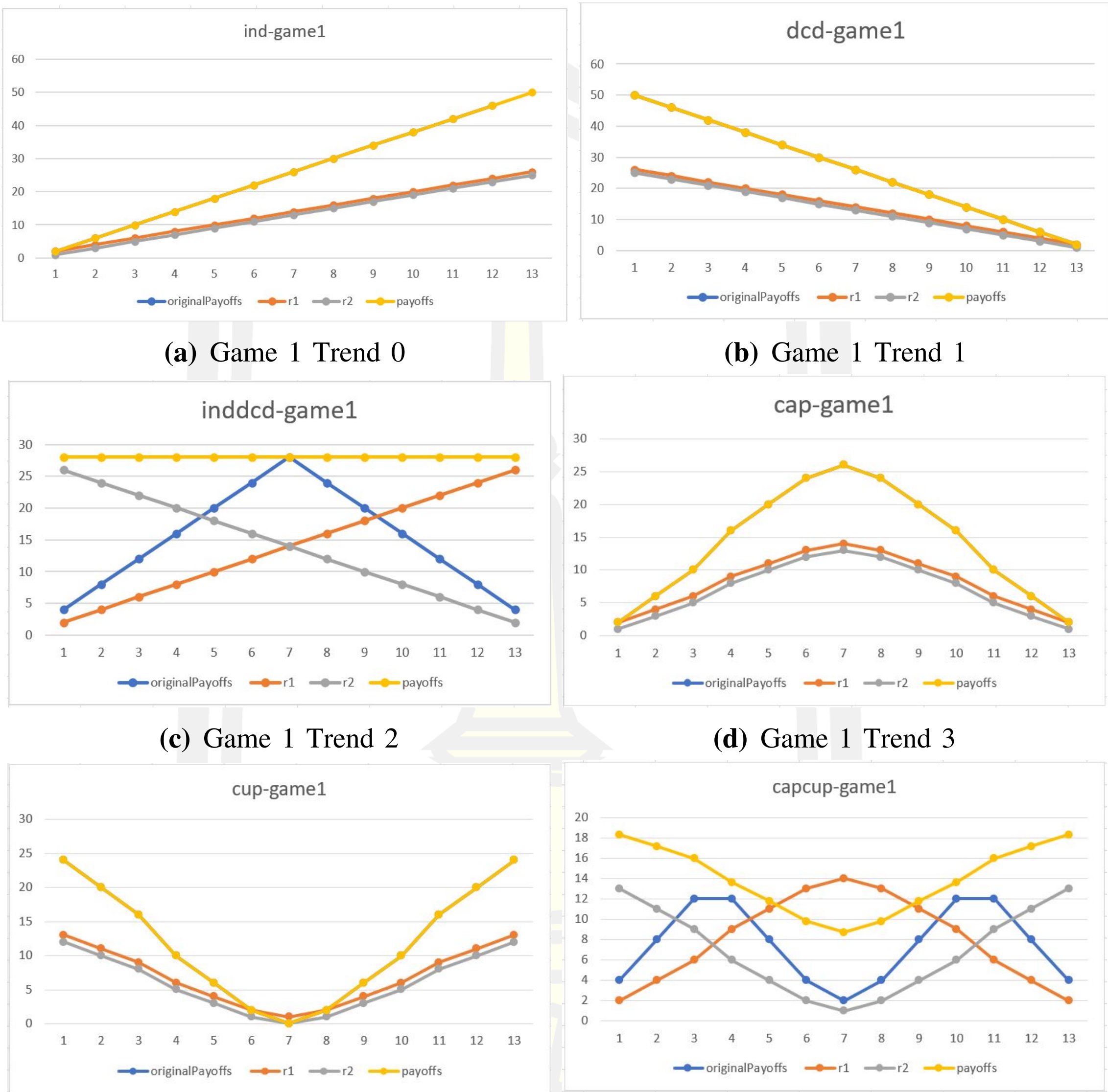
Case 6: Superadditive CAP VS CUP

In CAP VS CUP agent 1 and 13 have the lowest number of R_1 but have the highest number of R_2. Where as agent 8 has the highest number of R_2 but has the lowest number of R_2.

Table. 4.8. Game 1 and 2: Cap VS Cup Setting

		Agents												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Resource	**R1**	2	4	6	9	11	13	14	13	11	9	6	4	2
	R2	13	11	9	6	4	2	1	2	4	6	9	11	13

4.5 Experiment Results



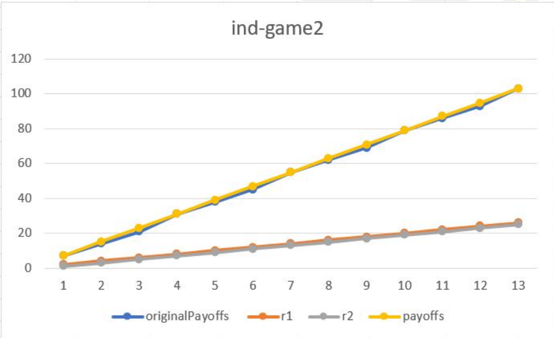
(a) Game 1 Trend 0 **(b)** Game 1 Trend 1

(c) Game 1 Trend 2 **(d)** Game 1 Trend 3

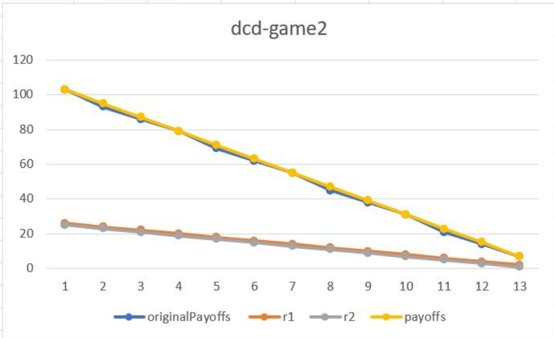
(e) Game 1 Trend 4 **(f)** Game 1 Trend 5

Figure. 4.2. Results Game 1 Trend 0-5

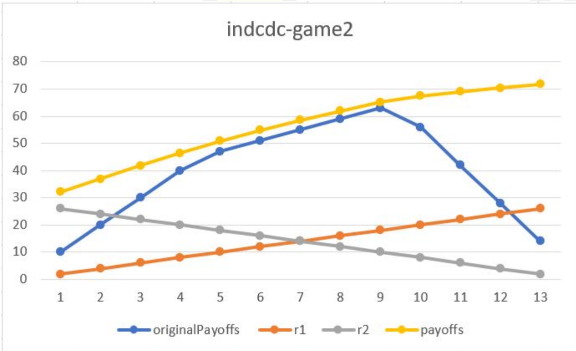
Figure 4.2 (a) shows that, results of original payoffs and payoffs of 13 agents in game 1 trend 0 continuous increase. Figure 4.2 (b) shows that, results of original payoffs and payoffs of 13 agents in game 1 trend 1 continuous decrease. Figure 4.2 (c) shows that, results of original payoffs are the rise and reach and payoffs are increased equally of 13 agents in game 1 trend 2. Figure 4.2 (d) shows that, results of original payoffs and payoffs are the rise and reach the peals for agent 1 then decrease and reach the lowest for agent 13 in game 1 trend 3. Figure 4.2 (e) shows that, results of original payoffs and payoffs are agent 1 and 13 have the highest number and whereas agent 7 has the lowest in game 1 trend 4. Figure 4.2 (f) shows that, the original payoffs of small (1 agent), medium (7 agents), and large (13 agents) coalitions are low. The payoffs of quite small (3, 4 agents) and quite (10, 11 agents) coalitions are high.



(a) Game 2 Trend 0



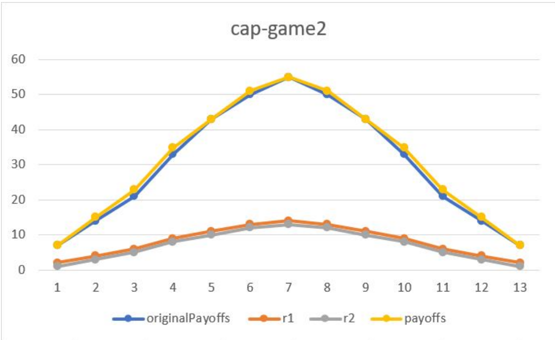
(b) Game 2 Trend 1



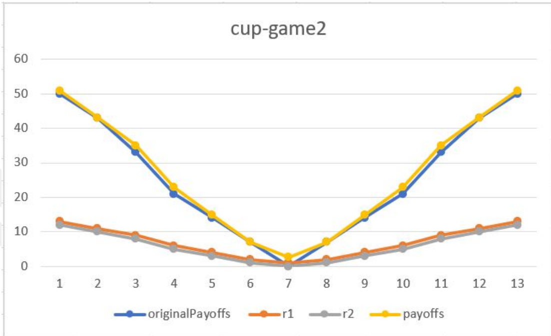
(c) Game 2 Trend 2

Figure. 4.3. Results Game 1 Trend 5 and Game2 Trend 0-2

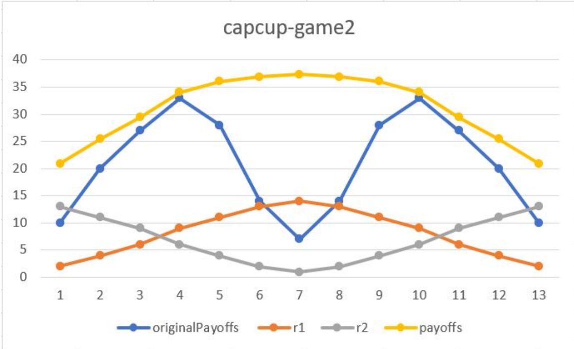
Figure 4.3 (a) shows that, the original payoffs of small (1 agent), medium (7 agents), and large (13 agents) coalitions are high. The payoffs of quite small (1 agents) and quite (10, 11 agents) coalitions are high. Figure 4.3 (b) shows that, the original payoffs of small (1 agent), medium (7 agents), and large (9 agents) coalitions are low. The payoffs of quite small (3, 4 agents) and quite (10, 11 agents) coalitions are low. Figure 4.3 (c) shows that, the original payoffs of small (1 agent), medium (7 agents), and large (9 agents) coalitions are low. The payoffs of quite small (3, 4 agents) and quite (10, 11 agents) coalitions are high.



(a) Game 2 Trend 3



(b) Game 2 Trend 4



(c) Game 2 Trend 5

Figure. 4.4. Results Game 2 Trend 3-5

Figure 4.4 (a) shows that, the original payoffs of small (1 agent), medium (7 agents), and large (13 agents) coalitions are low. The payoffs of quite small (3, 4 agents) and quite (10, 11 agents) coalitions are high. Figure 4.4 (b) shows that, the original payoffs of small (1 agent), medium (7 agents), and large (13 agents) coalitions are high. The payoffs of quite small (3, 4 agents) and quite (10, 11 agents) coalitions are high. Figure 4.4 (c) shows that, the original payoffs of small (1 agent), medium (7 agents), and large (9 agents) coalitions are low. The original payoffs of quite small (3, 4 agents) and quite (10, 11 agents) coalitions are low.

4.6 Conclusion

Based on [40, 59] where only certain games are considered, we found that with our setting where resources are given of agents in various trends, payoffs of agents differ from trends of resources. In other words, the main factor that controls the trend of agents payoffs are both technology matrix and trend of resources.

CHAPTER 5

FAIR PAYOFF IN BAKERY GAME IN NON-COOPERATIVE GAME

Strategic form game (SFG), a popular non-cooperative game theory, offer wide applications in real world. Generally, researchers use certain actions and corresponding payoffs to study SFG to find out the outcome of the game, which specifies how agents will behave and what the payoffs will be. Furthermore, traditional research in SFG considers merely actions and payoffs of agents. In real world, such information may not be known a priori but must be computed on the fly in timely fashion to be further used in online analytical processes. Here, we consider a more realistic environment, where payoffs are to be optimally computed from given resources and be used by agent for making decision. We are interested in wider spectrum of outcomes in games, where payoffs can vary within a trend such that the agents' strategies remain unchanged. We choose Bakery Game as our test bed to investigate two objectives: i) Explore range of resources and payoffs that does not affect the behaviors of agents. In other words, the payoffs remain fair to agents. and ii) Take into account whether computation of payoffs, using related factors, including resources, technology matrix, price function, and costs, is acceptable. The results show that there exist certain ranges of resources that agents do not change there strategies. Hence, agents receive fair payoffs. Furthermore, taking into account additional computations normally take place in real world environment do not affect the acceptable computation time for agents payoffs.

5.1 Setting

Let there be 3 cheese shops, a_1, a_2 and a_3. The resources needed for making cheese are mixed fruit topping, r_1, and cream cheese, r_2. Each of them has resources as following: $\{3, 30\}$, $\{11, 10\}$ and $\{6, 20\}$, respectively. It is known to all of them that there are two recipes for making cheese during festive Christmas. The first one is for making the top-quality cheese, g_1, each lb of which requires 0.2 lb of mixed fruit topping and 0.8 lb of cream cheese. The second one is for making the mid-quality cheese, g_2, each lb of which requires 0.2 lb of mixed fruit topping and 0.3 lb of cream cheese. Given this information, agents may form coalitions to pull resources and seek for higher benefits. Table 5.1 shows all possible coalitions and their resources.

Table. 5.1. Resources of agent coalitions

Coalition	a_1		a_2		a_3		Total	
	r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2
$\{a_1\}$	3	30	0	0	0	0	3	30
$\{a_2\}$	0	0	11	10	0	0	11	10
$\{a_3\}$	0	0	0	0	6	20	6	20
$\{a_1, a_2\}$	3	30	11	10	0	0	14	40
$\{a_1, a_3\}$	3	30	0	0	6	20	9	50
$\{a_2, a_3\}$	0	0	11	10	6	20	17	30
$\{a_1, a_2, a_3\}$	3	30	11	10	6	20	20	60

5.2 Prices

From previous seasons, it is also known to every agent that over supply can be costly. The demand and price relation for both goods is as the following:

$$d_1 = 190 - 25p_1 \qquad \text{and} \qquad d_2 = 250 - 50p_2,$$

where d_j is the demand (in pound) of g_j, and p_j is its price (in dollars per pound). The prices of both goods drops when the numbers are increased as shown in Figure 5.1. It is very important to note that this is common in real world and bring along a remarkable consequence, the profit, to the outcome of the game. The highest prices for both goods will be achieved if the number of goods are minimal, i.e. 1. For the top-quality cheese, the highest price is 7.56 per unit when it is produced only 1. For the mid-quality cheese, the highest price is 4.98 per unit when it is produced only 1. Both prices drop constantly when the number of each good is increased. The price of the top-quality cheese is 0 when the number of biscuit is 190. The price of the mid-quality cheese is 0 when the number of soft bread is 250.

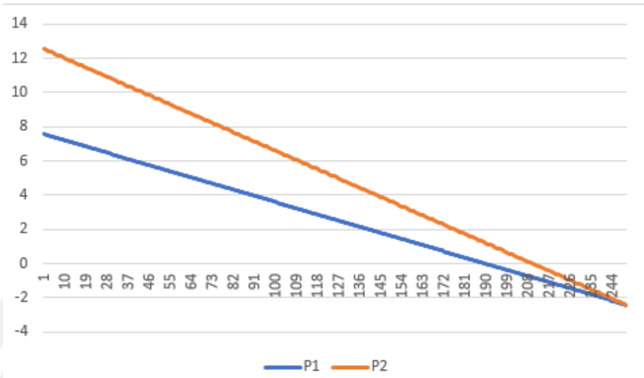


Figure. 5.1. Prices of the goods drop when the numbers are increased.

5.2.1 Profits

The objective function is to maximize profit

$$z = p_1x_1 + p_2x_2 \tag{5.1}$$

The product is sold out when the production does not exceed the requirements, i.e. $x_1 \leq D_1$ and $x_2 \leq D_2$. This implies that

$$x_1 + 25p_1 \leq 190 \qquad \text{and} \qquad x_2 + 50p_2 \leq 250. \tag{5.2}$$

The objective of the shops is to maximize their profit:

$$z = (7.6 - 0.04x_1)x_1 + (5 - 0.02x_2)x_2 \tag{5.3}$$

The profit is affected significantly by price function. As show in Figure 5.2, each good's profit increases slowly when it is close to the optimal point and drops after that. Given the price functions, the individual profits of goods are concave. The maximal profit of g_1 is 361 when the number of g_1 is 95 and the unit price is 3.8. The maximal profit of g_2 is 312.5 when the number of g_2 is 125 and the unit price is 2.5. After these points, both profits drop and reach 0 at $g_1 = 190$ and $g_2 = 250$, respectively. The direct implication is that both goods must not be produced beyond their optimal points (not necessarily the highest point). We can obviously see that there are incentives for agents to cooperate by pooling their resources to produce more goods. However, producing too many goods to the market can harm them, as we have just discussed.

The optimal global profit of producing good to the market is $g_1 = 95$ $p_1 = 3.8$ and $g_2 = 125$ $p_2 = 2.5$ yield the maximal profit of 673.5.

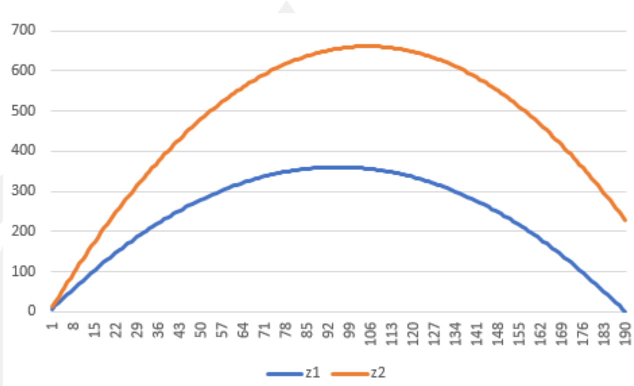


Figure. 5.2. Price functions and optimal plans of both goods.

The situation is more complex when an agent can produce both goods at the same time. Optimal plan for producing both goods, given enough resources, is shown in Figure 5.3.

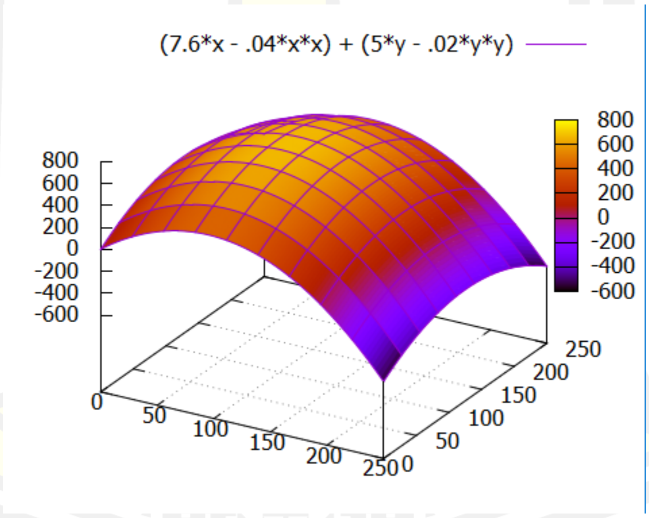


Figure. 5.3. Optimal plan for producing both goods.

5.3 Strategies and Outcomes

In a typical linear production problem, we can simply find the optimal plan for any agent, taking into account their resource constraints. However, the situation is far more complex here because there are 3 agents and strategies of other agents outside a coalition affect its value. Agents may form coalitions when their individual payoffs

are better than their singleton coalition values. The final profit of each agent also depends on other agents' strategies. If they greedily produce the same good because of its high price into the market, they will definitely suffer because the price will drop gradually. Here, there are 5 coalition structures (5 ways for agents to cooperate), i.e.
1) $\{\{a_1\}, \{a_2\}, \{a_3\}\}$,
2) $\{\{a_1, a_2\}, \{a_3\}\}$,
3) $\{\{a_1, a_3\}, \{a_2\}\}$,
4) $\{\{a_1\}, \{a_2, a_3\}\}$ and
5) $\{\{a_1, a_2, a_3\}\}$.

In order to find the outcome of this game, i.e. what coalition structure will take place, we need to know what will happen in each of them.

5.3.1 Case 1

We now consider case 1). Firstly, we find out what each agent can do best. There are many options for each agent. For example, agent a_1 may produce 15 units of g_1, a combination of 14 unit of g_1 plus 1 unit of g_2, ..., and 15 units of g_2. Among many possibilities, agents may have following plans. For a_1, strategy $s_{1,1}$ is to produce $g_1 = 15, g_2 = 0$, expecting profit of 105, strategy $s_{1,2}$ is to produce $g_1 = 8, g_2 = 7$ expecting profit of 92.26, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 12, g_2 = 0$ expecting profit of 85.44, strategy $s_{2,2}$ is to produce $g_1 = 9, g_2 = 9$ expecting profit of 108.54, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 33$ expecting profit of 143. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{3,2}$ is to produce $g_1 = 15, g_2 = 15$ expecting profit of 175.5, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 30$ expecting profit of 132. Unfortunately, the above expectations are not the final outcomes due to the fact that the actual prices depends on the number of goods being produced into the market. The higher the number, the lower the price.

Table. 5.2. Strategies of $\{a_1\}$, $\{a_2\}$, $\{a_3\}$ agent

$s_{1,1}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	82.8, 66.24, 138	88.8, 71.04, 88.8	97.8, 78.24, 0
	$s_{2,2}$	84.6, 50.76, 141	94.8, 56.88, 94.8	99.6, 59.76, 0
	$s_{2,3}$	90, 0, 150	96, 0, 96	105, 0, 0

$s_{1,2}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	46.4, 69.6, 145	49.6, 74.4, 93	54.4, 81.6, 0
	$s_{2,2}$	47.36, 53.28, 148	50.56, 56.88, 94.8	55.36, 62.28, 0
	$s_{2,3}$	50.24, 0, 157	53.44, 0, 100.2	58.24, 0, 0

$s_{1,3}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	0, 73.44, 153	0, 78.24, 97.8	0, 85.44, 0
	$s_{2,2}$	0, 56.16, 156	0, 59.76, 99.6	0, 65.16, 0
	$s_{2,3}$	0, 0, 165	0, 0, 100.2	0, 0, 0

We plot agents' strategies and respective outcomes in table 5.2. The first outcome (82.8, 66.24, 138) means agent a_1 gains 82.8 by producing 15 units of g_1, agent a_2 gains 66.24 by producing 12 units of g_1 and agent a_3 gains 138 by producing 25 units of g_1. This is because the total number of g_1 is 52 lowering the unit price to 5.52. We now focus on agent a_3. Suppose, agent a_1 plays $s_{1,1}$. If agent a_2 plays $s_{2,1}$, the best strategy for a_3 is $s_{3,1}$ because it gives the maximal payoff 138 to a_3. This is also the case even a_2 chose to play $s_{2,2}$ or $s_{2,3}$. This strategy $s_{1,3}$ is also the best choice for a_3 if a_1 plays $s_{1,2}$, regardless of what strategy a_2 plays, or $s_{1,3}$, regardless of what strategy a_2 plays. We can conclude that in case 1), where agents do not cooperate, agent a_3 will always play $s_{1,3}$. Following the same analytic process, the outcome of case 1) is a_1 plays $s_{1,1}$ and gains 82.8, a_2 plays $s_{2,1}$ and gains 66.24, and a_3 plays $s_{3,1}$ and gains 138.

5.3.2 Case 2, 3, 4 and 5

We now consider case 2, in which the coalitions are $\{a_1, a_2\}$ and $\{a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_2\}$, strategy $s_{12,1}$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 280, strategy $s_{12,2}$ is to produce $g_1 = 35, g_2 = 34$ expecting profit of 363.88, strategy $s_{12,3}$ is to produce $g_1 = 10, g_2 = 60$ expecting profit of 300. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{3,2}$ is to produce $g_1 = 15, g_2 = 15$ expecting profit of 175.5, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 30$ expecting profit of 132. We plot agents' strategies and respective outcomes in table 5.3.

Table. 5.3. Strategies of $\{a_1, a_2\}$, $\{a_3\}$ agent

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1, a_2	$s_{12,1}$	230, 115	250, 75	280, 0
	$s_{12,2}$	182, 130	196, 84	217, 0
	$s_{12,3}$	62, 155	66, 99	72, 0

Following the same analytic process, the outcome of case 2 is $\{a_1, a_2\}$ play $s_{12,1}$ and gain 230, and $\{a_3\}$ plays $s_{3,1}$ and gains 115.

We now consider case 3, in which the coalitions are $\{a_1, a_3\}$ and $\{a_2\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_3\}$, strategy $s_{13,1}$ is to produce $g_1 = 44, g_2 = 0$, expecting profit of 256.96, strategy $s_{13,2}$ is to produce $g_1 = 23, g_2 = 22$ expecting profit of 253.96, strategy $s_{13,3}$ is to produce $g_1 = 0, g_2 = 45$ expecting profit of 184.5. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 12, g_2 = 0$ expecting profit of 85.44, strategy $s_{2,2}$ is to produce $g_1 = 9, g_2 = 9$ expecting profit of 108.54, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 33$ expecting profit of 143. We plot agents' strategies and respective outcomes in table 5.4.

Table. 5.4. Strategies of $\{a_1, a_3\}$, $\{a_2\}$ agent

		a_2		
		$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
a_1, a_3	$s_{13,1}$	235.84, 64.32	241.12, 49.32	256.96, 0
	$s_{13,2}$	142.6, 74.4	145.36, 56.88	153.64, 0
	$s_{13,3}$	0, 85.44	0, 65.16	0, 0

Following the same analytic process, the outcome of case 3 is $\{a_1, a_3\}$ play $s_{13,1}$ and gain 235.84, and $\{a_2\}$ plays $s_{2,1}$ and gains 64.32

We now consider case 4, in which the coalitions are $\{a_1\}$ and $\{a_2, a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1\}$, strategy $s_{1,1}$ is to produce $g_1 = 15, g_2 = 0$, expecting profit of 105, strategy $s_{1,2}$ is to produce $g_1 = 8, g_2 = 7$ expecting profit of 92.26, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. For a_2, a_3, strategy $s_{23,1}$ is to produce $g_1 = 37, g_2 = 0$ expecting profit of 226.44, strategy $s_{23,2}$ is to produce $g_1 = 10, g_2 = 10$ expecting profit of 120, strategy $s_{23,3}$ is to produce $g_1 = 0, g_2 = 85$ expecting profit of 280.5. We plot agents' strategies and respective outcomes in table 5.5.

Table. 5.5. Strategies of $\{a_1\}$, $\{a_2, a_3\}$ agent

		a_2, a_3		
		$s_{23,1}$	$s_{23,2}$	$s_{23,3}$
a_1	$s_{1,1}$	82.8, 204.24	99, 66	105, 0
	$s_{1,2}$	46.4, 214.6	55.04, 68.8	58.24, 0
	$s_{1,3}$	0, 226.44	0, 72	0, 0

Following the same analytic process, the outcome of case 4 is $\{a_1\}$ play $s_{1,1}$ and gain 82.8, and $\{a_2, a_3\}$ plays $s_{23,1}$ and gains 204.24

Lastly, the grand coalition $\{a_1, a_2, a_3\}$ can produce 55 unit of g_1 and 45 units of g_2 and make profit of 481.

5.3.3 Payoffs by Shapley Value

As previously mentioned, this is more complicated than the environment Shapley value was originally assumed for. Because strategies of agents outside a coalition can affect the coalition value, we need to take the affected coalition values to compute the payoffs for agents. By taking into account other agents' strategies, the affected coalition values vary. According to the basic principle of cooperative game, the coalition value is the highest possible value the coalition can make. Therefore, the coalition values for $\{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}$ and $\{a_1, a_2, a_3\}$ are 82.8, 66.24, 138, 230, 235.48, 204.24 and 481, respectively. the payoffs for agents are 178.11, 139.49,163.39.

If we do not take into account the strategies of agents outside coalitions, the coalition values will be 105, 143, 186, 370, 265.16, 333 and 481. The (mistakenly calculated) payoffs will be 157.36, 188.28 and 135.36, which are very different from the correct ones.

5.4 Experiments

In this section, we explore further to see the behavior of Shapley value on payoffs for agents in the aforementioned bakery game.

5.4.1 Settings

We consider game of 3 agents, similarly to the previous section, but with more varieties on the agents' possession. The sum of r_1 is 30 and the sum of r_2 is 60. Both resources are distributed among agent in five trends. In trend 1, both goods are distributed evenly to all agents, i.e. 10 units of r_1 and 20 units of r_2. In trend 2, a_1 receives merely 3 units of r_1 but 35 units of r_2, a_2 receives the same number of resources as in trend 1 and also in other trends, and a_3 receives 35 units of r_1 and 3 units of r_2. We can see that the number of resources of a_1 and a_2 are opposite. Trend 3 is a reverse of trend 2. In trend 4, a_1 receives low number of both resources, while a_2 receives high number of them. Trend 5 is a reverse of trend 4. The resources distributed to agents are shown in table 5.6.

Table. 5.6. Ressource

Trends	Resources					
	a_1		a_2		a_3	
	r_1	r_2	r_1	r_2	r_1	r_2
1	10	20	10	20	10	20
2	3	35	10	20	17	5
3	17	5	10	20	3	35
4	3	5	10	20	17	35
5	17	35	10	20	3	5

5.5 Strategies and Outcomes

By following the complex analytic processes, we receive the outcomes of each trend as shown in table 5.7. The outstanding figure is the grand coalition's value remain the same and is the best strategy for all agents. Another notable result is that a_2's coalition values change all the times even it receives the same number of resources in all trends. Lastly, trend 2 and 3 are diagonally similar as well as trend 4 and 5.

Table. 5.7. Coalition Structure(CS)

Trends	Outcomes				
	$\{a_1\},\{a_2\},\{a_3\}$	$\{a_1,a_2\},\{a_3\}$	$\{a_1,a_3\},\{a_2\}$	$\{a_1\},\{a_2,a_3\}$	$\{a_1,a_2,a_3\}$
1	$s_{1,1} = [25,0] = 115$, $s_{2,1} = [25,0] = 115$, $s_{3,1} = [25,0] = 115$	$s_{12,1} = [50,0] = 230$, $s_{3,1} = [25,0] = 115$	$s_{13,1} = [50,0] = 230$, $s_{2,1} = [25,0] = 115$	$s_{1,1} = [25,0] = 115$, $s_{23,1} = [50,0] = 230$	533.76
2	$s_{1,1} = [15,0] = 86.4$, $s_{2,1} = [25,0] = 144$, $s_{3,1} = [6,0] = 34.56$	$s_{12,1} = [65,0] = 309.4$, $s_{3,1} = [6,0] = 28.56$	$s_{13,1} = [50,0] = 230$, $s_{2,1} = [25,0] = 115$	$s_{1,1} = [15,0] = 86.4$, $s_{23,1} = [31,0] = 178.56$	533.76
3	$s_{1,1} = [6,0] = 34.56$, $s_{2,1} = [25,0] = 144$, $s_{3,1} = [15,0] = 86.4$	$s_{12,1} = [31,0] = 178.56$, $s_{3,1} = [15,0] = 86.4$	$s_{13,1} = [50,0] = 230$, $s_{2,1} = [25,0] = 115$	$s_{1,1} = [6,0] = 28.56$, $s_{23,1} = [65,0] = 309.4$	533.76
4	$s_{1,1} = [6,0] = 27.84$, $s_{2,1} = [25,0] = 116$, $s_{3,1} = [43,2] = 199.52$	$s_{12,1} = [31,0] = 143.84$, $s_{3,1} = [43,2] = 199.52$	$s_{13,1} = [50,0] = 230$, $s_{2,1} = [25,0] = 115$	$s_{1,1} = [6,0] = 27.84$, $s_{23,1} = [68,1] = 315.52$	533.76
5	$s_1, s_1 = [43,2] = 199.52$, $s_{2,1} = [25,0] = 116$, $s_{3,1} = [6,0] = 27.84$	$s_{12,1} = [68,1] = 315.52$, $s_{3,1} = [6,0] = 27.84$	$s_{13,1} = [50,0] = 230$, $s_{2,1} = [25,0] = 115$	$s_{1,1} = [43,2] = 199.52$, $s_{23,1} = [31,0] = 143.84$	533.76

The agents' payoffs are shown in table 5.28. As one may expect, the agents' payoffs have some pattern reflecting the patterns of resources the are allocated. In trend 2 and 3, the payoffs of a_1 and a_3 are diagonally similar, i.e. 116 and 207.34, while a_2 receives 210.42. In trend 4 and 5, the payoffs of a_1 and a_3 are also diagonally similar, i.e. 263.43 and 91.75, while a_2 receives 178.59.

5.5.1 Trend 1

case 1

For a_1, strategy $s_{1,1}$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 165, strategy $s_{1,2}$ is to produce $g_1 = 18, g_2 = 17$ expecting profit of 203.06, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 18, g_2 = 17$ expecting profit of 203.06, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{3,2}$ is to produce $g_1 = 18, g_2 = 17$ expecting profit of 203.06, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. Unfortunately, the above expectations are not the final outcomes due to the fact that the actual prices depends on the number of goods being produced into the market. The higher the number, the lower the price.

We plot agents' strategies and respective outcomes in table 5.8. The first outcome (115, 115, 115) means agent a_1 gains 115 by producing 25 units of g_1, agent a_2 gains 115 by producing 25 units of g_1 and agent a_3 gains 115 by producing 25 units of g_1. This is because the total number of g_1 is 75 lowering the unit price to 4.6. We now focus on agent a_3. Suppose, agent a_1 plays $s_{1,1}$. If agent a_2 plays $s_{2,1}$, the best strategy for a_3 is $s_{3,1}$ because it gives the maximal payoff 115 to a_3. This is also the case even a_2 chose to play $s_{2,2}$ or $s_{2,3}$. This strategy $s_{1,3}$ is also the best choice for a_3 if a_1 plays $s_{1,2}$, regardless of what strategy a_2 plays, or $s_{1,3}$, regardless of what strategy a_2 plays. We can conclude that in case 1), where agents do not cooperate, agent a_3 will always play $s_{1,3}$. Following the same analytic process, the outcome of case 1) is a_1 plays $s_{1,1}$ and gains 115, a_2 plays $s_{2,1}$ and gains 115, and a_3 plays $s_{3,1}$ and gains 115.

Table. 5.8. Strategies of $\{a_1\}$, $\{a_2\}$, $\{a_3\}$ agent

$s_{1,1}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	115, 115, 115	122, 122, 87.84	140, 140, 0
	$s_{2,2}$	122, 87.84, 122	129, 92.88, 92.88	147, 105.84, 0
	$s_{2,3}$	140, 0, 140	147, 0, 105.84	165, 0, 0

$s_{1,2}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	87.84, 122, 122	92.88, 129, 92.88	113.04, 157, 0
	$s_{2,2}$	92.88, 92.88, 129	97.92, 97.92, 97.92	110.88, 110.88, 0
	$s_{2,3}$	105.84, 0, 147	110.88, 0, 110.88	123.84, 0, 0

$s_{1,3}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	0, 140, 140	0, 147, 105.84	0, 165, 0
	$s_{2,2}$	0, 105.84, 147	0, 110.88, 110.88	0, 123.84, 0
	$s_{2,3}$	0, 0, 165	0, 0, 123.84	0, 0, 0

Case 2, 3, 4 and 5

We now consider case 2, in which the coalitions are $\{a_1, a_2\}$ and $\{a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_2\}$, strategy $s_{12,1}$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 280, strategy $s_{12,2}$ is to produce $g_1 = 35, g_2 = 34$ expecting profit of 363.88, strategy $s_{12,3}$ is to produce $g_1 = 0, g_2 = 50$ expecting profit of 200. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{3,2}$ is to produce $g_1 = 18, g_2 = 17$ expecting profit of 203.06, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. We plot agents' strategies and respective outcomes in table 5.9.

Table. 5.9. Strategies of $\{a_1, a_2\}$, $\{a_3\}$ agent

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1, a_2	$s_{12,1}$	230, 115	244, 87.84	280, 0
	$s_{12,2}$	182, 130	191.8, 98.64	217, 0
	$s_{12,3}$	0, 165	0, 123.84	0, 0

Following the same analytic process, the outcome of case 2 is $\{a_1, a_2\}$ play $s_{12,1}$

and gain 230, and $\{a_3\}$ plays $s_{3,1}$ and gains 115.

We now consider case 3, in which the coalitions are $\{a_1, a_3\}$ and $\{a_2\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_3\}$, strategy $s13, 1$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 280, strategy $s_{13,2}$ is to produce $g_1 = 15, g_2 = 14$ expecting profit of 171.08, strategy $s_{13,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 200. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 18, g_2 = 17$ expecting profit of 203.06, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. We plot agents' strategies and respective outcomes in table 5.10.

Table. 5.10. Strategies of $\{a_1, a_3\}$, $\{a_2\}$ agent

		a_2		
		$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
a_1, a_3	$s_{13,1}$	230, 115	244, 87.84	280, 0
	$s_{13,2}$	90, 150	94.2, 113.04	105, 0
	$s_{13,3}$	0, 165	0, 123.84	0, 0

Following the same analytic process, the outcome of case 3 is $\{a_1, a_3\}$ play $S_{13,1}$ and gain 230, and $\{a_2\}$ plays $S_{2,1}$ and gains 115

We now consider case 4, in which the coalitions are $\{a_1\}$ and $\{a_2, a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1\}$, strategy $s_{1,1}$ is to produce $g_1 = 25, g_2 = 0$, expecting profit of 165, strategy $s_{1,2}$ is to produce $g_1 = 18, g_2 = 17$ expecting profit of 203.06, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. For a_2, a_3, strategy $s_{23,1}$ is to produce $g_1 = 50, g_2 = 0$ expecting profit of 280, strategy $s_{23,2}$ is to produce $g_1 = 25, g_2 = 24$ expecting profit of 273.48, strategy $s_{23,3}$ is to produce $g_1 = 12, g_2 = 88$ expecting profit of 370.56. We plot agents' strategies and respective outcomes in table 5.11.

Table. 5.11. Strategies of $\{a_1\}$, $\{a_2, a_3\}$ agent

		a_2, a_3		
		$s_{23,1}$	$s_{23,2}$	$s_{23,3}$
a_1	$s_{1,1}$	115, 230	140, 140	153, 73.44
	$s_{1,2}$	87.84, 244	105.84, 147	115.2, 76.8
	$s_{1,3}$	0, 280	0, 165	0, 85.44

Following the same analytic process, the outcome of case 4 is $\{a_1\}$ play $s_{1,1}$ and gain 115, and $\{a_2, a_3\}$ plays $s_{23,1}$ and gains 230

5.5.2 Trend 2

Case 1

For a_1, strategy $s_{1,1}$ is to produce $g_1 = 15, g_2 = 0$, expecting profit of 105, strategy $s_{1,2}$ is to produce $g_1 = 8, g_2 = 7$ expecting profit of 92.26, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 13, g_2 = 12$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 6, g_2 = 0$ expecting profit of 44.16, strategy $s_{3,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 16$ expecting profit of 47.88. Unfortunately, the above expectations are not the final outcomes due to the fact that the actual prices depends on the number of goods being produced into the market. The higher the number, the lower the price.

Table. 5.12. Strategies of $\{a_1\}$, $\{a_2\}$, $\{a_3\}$ agent

$s_{1,1}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	86.4, 144, 34.56	88.2, 147, 17.64	90, 150, 0
	$s_{2,2}$	93.6, 81.12, 37.44	95.4, 82.68, 19.08	97.2, 84.24, 0
	$s_{2,3}$	101.4, 0, 40.56	103.2, 0, 20.64	105, 0, 0

$s_{1,2}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	48.32, 151, 36.24	49.28, 154, 18.48	50.24, 157, 0
	$s_{2,2}$	52.16, 84.76, 39.12	53.12, 86.32, 19.92	54.08, 87.88, 0
	$s_{2,3}$	56.32, 0, 42.24	57.28, 0, 21.48	58.24, 0, 0

$s_{1,3}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	0, 159, 38.16	0, 162, 19.44	0, 165, 0
	$s_{2,2}$	0, 81.12, 37.44	0, 90.48, 20.88	0, 92.04, 0
	$s_{2,3}$	0, 0, 44.16	0, 0, 22.44	0, 0, 0

We plot agents' strategies and respective outcomes in table 5.12. The first outcome (86.4, 144, 34.56) means agent a_1 gains 86.4 by producing 15 units of g_1, agent a_2

gains 144 by producing 25 units of g_1 and agent a_3 gains 34.56 by producing 6 units of g_1. This is because the total number of g_1 is 46 lowering the unit price to 5.76. We now focus on agent a_3. Suppose, agent a_1 plays $s_{1,1}$. If agent a_2 plays $s_{2,1}$, the best strategy for a_3 is $s_{3,1}$ because it gives the maximal payoff 34.56 to a_3. This is also the case even a_2 chose to play $s_{2,2}$ or $s_{2,3}$. This strategy $s_{1,3}$ is also the best choice for a_3 if a_1 plays $s_{1,2}$, regardless of what strategy a_2 plays, or $s_{1,3}$, regardless of what strategy a_2 plays. We can conclude that in case 1), where agents do not cooperate, agent a_3 will always play $s_{1,3}$. Following the same analytic process, the outcome of case 1) is a_1 plays $s_{1,1}$ and gains 86.4, a_2 plays $s_{2,1}$ and gains 144, and a_3 plays $s_{3,1}$ and gains 34.56.

Case 2, 3, 4 and 5

We now consider case 2, in which the coalitions are $\{a_1, a_2\}$ and $\{a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_2\}$, strategy $s_{12,1}$ is to produce $g_1 = 65, g_2 = 0$, expecting profit of 325, strategy $s_{12,2}$ is to produce $g_1 = 33, g_2 = 32$ expecting profit of 346.76, strategy $s_{12,3}$ is to produce $g_1 = 12, g_2 = 52$ expecting profit of 291.36. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 6, g_2 = 0$ expecting profit of 44.16, strategy $s_{3,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 16$ expecting profit of 74.88. We plot agents' strategies and respective outcomes in table 5.13.

Table. 5.13. Strategies of $\{a_1, a_2\}$, $\{a_3\}$ agent

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1, a_2	$s_{12,1}$	309.4, 28.56	317.2, 14.64	494, 0
	$s_{12,2}$	199.32, 36.24	203.28, 18.48	207.24, 0
	$s_{12,3}$	0, 41.28	84, 21	85.44, 0

Following the same analytic process, the outcome of case 2 is $\{a_1, a_2\}$ play $s_{12,1}$ and gain 309.4, and $\{a_3\}$ plays $s_{3,1}$ and gains 28.56.

We now consider case 3, in which the coalitions are $\{a_1, a_3\}$ and $\{a_2\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_3\}$, strategy $s_{13,1}$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 280, strategy $s_{13,2}$ is to produce $g_1 = 25, g_2 = 24$ expecting profit of 273.48, strategy $s_{13,3}$ is to produce $g_1 = 13, g_2 = 86$ expecting profit of 374.12. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 18, g_2 = 17$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. We plot agents' strategies and respective outcomes in table 5.14.

Table. 5.14. Strategies of $\{a_1, a_3\}$, $\{a_2\}$ agent

		a_2		
		$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
a_1, a_3	$s_{13,1}$	230, 115	254, 66.04	280, 0
	$s_{13,2}$	140, 140	91.2, 79.04	165, 0
	$s_{13,3}$	79.04, 152	85.28, 85.28	92.04, 0

Following the same analytic process, the outcome of case 3 is $\{a_1, a_3\}$ play $s_{13,1}$ and gain 230, and $\{a_2\}$ plays $s_{2,1}$ and gains 115

We now consider case 4, in which the coalitions are $\{a_1\}$ and $\{a_2, a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1\}$, strategy $s_{1,1}$ is to produce $g_1 = 15, g_2 = 0$, expecting profit of 105, strategy $s_{1,2}$ is to produce $g_1 = 8, g_2 = 7$ expecting profit of 92.26, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. For a_2, a_3, strategy $s_{23,1}$ is to produce $g_1 = 31, g_2 = 0$ expecting profit of 197.16, strategy $s_{23,2}$ is to produce $g_1 = 16, g_2 = 15$ expecting profit of 181.86, strategy $s_{23,3}$ is to produce $g_1 = 0, g_2 = 83$ expecting profit of 277.22. We plot agents' strategies and respective outcomes in table 5.15.

Table. 5.15. Strategies of $\{a_1\}$, $\{a_2, a_3\}$ agent

		a_2, a_3		
		$s_{23,1}$	$s_{23,2}$	$s_{23,3}$
a_1	$s_{1,1}$	86.4, 178.56	95.4, 101.76	105, 0
	$s_{1,2}$	48.32, 187.24	53.12, 106.24	58.24, 0
	$s_{1,3}$	0, 197.16	0, 111.36	0, 0

Following the same analytic process, the outcome of case 4 is $\{a_1\}$ play $s_{1,1}$ and gain 86.4, and $\{a_2, a_3\}$ plays $s_{23,1}$ and gains 178.56

5.5.3 Trend 3

Case 1

For a_1, strategy $s_{1,1}$ is to produce $g_1 = 6, g_2 = 0$, expecting profit of 44.16, strategy $s_{1,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 16$ expecting profit of 74.88. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 13, g_2 = 12$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 15, g_2 = 0$ expecting profit of 105, strategy $s_{3,2}$ is to produce $g_1 = 8, g_2 = 7$ expecting profit of 92.26, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. Unfortunately, the above expectations are not the final outcomes due to the fact that the actual prices depends on the number of goods being produced into the market. The higher the number, the lower the price.

Table. 5.16. Strategies of $\{a_1\}$, $\{a_2\}$, $\{a_3\}$ agent

$s_{1,1}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	34.56, 144, 86.4	36.24, 151, 48.32	38.16, 159, 0
	$s_{2,2}$	37.44, 81.12, 93.6	39.12, 84.76, 52.16	41.04, 88.92, 0
	$s_{2,3}$	40.56, 0, 101.4	42.24, 0, 56.32	44.16, 0, 0

$s_{1,2}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	17.64, 147, 88.2	18.48, 154, 49.28	19.44, 162, 0
	$s_{2,2}$	19.08, 82.68, 95.4	19.92, 86.32, 53.12	20.88, 90.48, 0
	$s_{2,3}$	20.64, 0, 103.2	21.48, 0, 57.28	22.44, 0, 0

$s_{1,3}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	0, 150, 90	0, 157, 50.24	0, 165, 0
	$s_{2,2}$	0, 84.24, 97.2	0, 87.88, 54.08	0, 92.04, 0
	$s_{2,3}$	0, 0, 105	0, 0, 58.24	0, 0, 0

We plot agents' strategies and respective outcomes in table 5.16. The first outcome (34.56, 144, 86.4) means agent a_1 gains 34.56 by producing 6 units of g_1, agent a_2 gains 144 by producing 25 units of g_1 and agent a_3 gains 86.4 by producing 15 units of g_1. This is because the total number of g_1 is 46 lowering the unit price to 5.76. We now focus on agent a_3. Suppose, agent a_1 plays $s_{1,1}$. If agent a_2 plays $s_{2,1}$, the best strategy for a_3 is $s_{3,1}$ because it gives the maximal payoff 34.56 to a_3. This is also the case even a_2 chose to play $s_{2,2}$ or $s_{2,3}$. This strategy $s_{1,3}$ is also the best choice for a_3 if a_1 plays $s_{1,2}$, regardless of what strategy a_2 plays, or $s_{1,3}$, regardless of what strategy a_2 plays. We can conclude that in case 1), where agents do not cooperate, agent a_3 will always play $s_{1,3}$. Following the same analytic process, the outcome of case 1) is a_1 plays $s_{1,1}$ and gains 34.56, a_2 plays $s_{2,1}$ and gains 144, and a_3 plays $s_{3,1}$ and gains 86.4.

Case 2, 3, 4 and 5

We now consider case 2, in which the coalitions are $\{a_1, a_2\}$ and $\{a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_2\}$, strategy $s_{12,1}$ is to produce $g_1 = 15, g_2 = 0$, expecting profit of 105, strategy $s_{12,2}$ is to produce $g_1 = 8, g_2 = 7$ expecting profit of 92.26, strategy $s_{12,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 31, g_2 = 0$ expecting profit of 197.16, strategy $s_{3,2}$ is to produce $g_1 = 15, g_2 = 15$ expecting profit of 181.86, strategy $s_{3,3}$ is to produce $g_1 = 83, g_2 = 83$ expecting profit of 277.22. We plot agents' strategies and respective outcomes in table 5.17.

Table. 5.17. Strategies of $\{a_1, a_2\}$, $\{a_3\}$ agent

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1, a_2	$s_{12,1}$	86.4, 178.56	95.4, 101.76	105, 0
	$s_{12,2}$	48.32, 187.24	53.12, 106.24	58.24, 0
	$s_{12,3}$	0, 197.16	0, 111.36	0, 0

Following the same analytic process, the outcome of case 2 is $\{a_1, a_2\}$ play $s_{12,1}$ and gain 86.4, and $\{a_3\}$ plays $s_{3,1}$ and gains 178.56.

We now consider case 3, in which the coalitions are $\{a_1, a_3\}$ and $\{a_2\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_3\}$, strategy $s_{13,1}$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 280, strategy $s_{13,2}$ is to produce $g_1 = 25, g_2 = 24$ expecting profit of 273.48, strategy $s_{13,3}$ is to produce $g_1 = 16, g_2 = 86$ expecting profit of 239.54. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 13, g_2 = 12$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 25$ expecting profit of 112.5. We plot agents' strategies and respective outcomes in table 5.18.

Table. 5.18. Strategies of $\{a_1, a_3\}$, $\{a_2\}$ agent

		a_2		
		$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
a_1, a_3	$s_{13,1}$	230, 115	254, 66.04	280, 0
	$s_{13,2}$	140, 140	91.2, 79.04	165, 0
	$s_{13,3}$	92.8, 168.2	103.04, 83.72	111.36, 0

Following the same analytic process, the outcome of case 3 is $\{a_1, a_3\}$ play $s_{13,1}$ and gain 230, and $\{a_2\}$ plays $s_{2,1}$ and gains 115

We now consider case 4, in which the coalitions are $\{a_1\}$ and $\{a_2, a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1\}$, strategy $s_{1,1}$ is to produce $g_1 = 6, g_2 = 0$, expecting profit of 44.16, strategy $s_{1,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 16$ expecting profit of 74.88. For a_2, a_3, strategy $s_{23,1}$ is to produce $g_1 = 65, g_2 = 0$ expecting profit of 325, strategy $s_{23,2}$ is to produce $g_1 = 33, g_2 = 32$ expecting profit of 346.76, strategy $s_{23,3}$ is to produce $g_1 = 12, g_2 = 52$ expecting profit of 291.36. We plot agents' strategies and respective outcomes in table 5.19.

Table. 5.19. Strategies of $\{a_1\}$, $\{a_2, a_3\}$ agent

		a_2, a_3		
		$s_{23,1}$	$s_{23,2}$	$s_{23,3}$
a_1	$s_{1,1}$	28.56, 309.4	36.24, 199.32	41.28, 82.56
	$s_{1,2}$	14.64, 317.2	18.48, 203.28	21, 84
	$s_{1,3}$	0, 325	0, 207.24	0, 85.44

Following the same analytic process, the outcome of case 4 is $\{a_1\}$ play $s_{1,1}$ and gain 28.56, and $\{a_2, a_3\}$ plays $s_{23,1}$ and gains 309.4

5.5.4 Trend 4

Case 1

For a_1, strategy $s_{1,1}$ is to produce $g_1 = 6, g_2 = 0$, expecting profit of 44.16, strategy $s_{1,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 13, g_2 = 12$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 50$ expecting profit of 200. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 43, g_2 = 2$ expecting profit of 262.76, strategy $s_{3,2}$ is to produce $g_1 = 22, g_2 = 21$ expecting profit of 244.02, strategy $s_{3,3}$ is to produce $g_1 = 11, g_2 = 74$ expecting profit of 339.24. Unfortunately, the above expectations are not the final outcomes due to the fact that the actual prices depends on the number of goods being produced into the market. The higher the number, the lower the price.

Table. 5.20. Strategies of $\{a_1\}$, $\{a_2\}$, $\{a_3\}$ agent

$s_{1,1}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	27.84, 116, 199.52	32.88, 137, 120.56	35.52, 148, 65.12
	$s_{2,2}$	30.72, 66.56, 220.16	35.76, 77.48, 131.12	38.4, 83.2, 70.4
	$s_{2,3}$	33.84, 0, 242.52	38.88, 0, 142.56	41.52, 0, 76.12

$s_{1,2}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	17.64, 147, 252.84	16.8, 140, 123.2	18.12, 151, 66.44
	$s_{2,2}$	15.12, 65.52, 216.72	18.24, 79.04, 133.76	19.56, 84.76, 71.72
	$s_{2,3}$	17.28, 0, 247.68	19.8, 0, 145.2	21.12, 0, 77.44

$s_{1,3}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	0, 122, 209.84	0, 143, 125.84	0, 154, 67.76
	$s_{2,2}$	0, 69.68, 230.48	0, 80.6, 136.4	0, 86.32, 73.04
	$s_{2,3}$	0, 0, 88.2	0, 0, 53.76	0, 0, 78.76

We plot agents' strategies and respective outcomes in table 5.20. The first outcome (27.84, 116, 199.52) means agent a_1 gains 27.84 by producing 6 units of g_1, agent a_2

gains 116 by producing 25 units of g_1 and agent a_3 gains 199.52 by producing 43 units of g_1. This is because the total number of g_1 is 74 lowering the unit price to 4.64. We now focus on agent a_3. Suppose, agent a_1 plays $s_{1,1}$. If agent a_2 plays $s_{2,1}$, the best strategy for a_3 is $s_{3,1}$ because it gives the maximal payoff 199.52 to a_3. This is also the case even a_2 chose to play $s_{2,2}$ or $s_{2,3}$. This strategy $s_{1,3}$ is also the best choice for a_3 if a_1 plays $s_{1,2}$, regardless of what strategy a_2 plays, or $s_{1,3}$, regardless of what strategy a_2 plays. We can conclude that in case 1), where agents do not cooperate, agent a_3 will always play $s_{1,3}$. Following the same analytic process, the outcome of case 1) is a_1 plays $s_{1,1}$ and gains 27.84, a_2 plays $s_{2,1}$ and gains 116, and a_3 plays $s_{3,1}$ and gains 199.52.

Case 2, 3, 4 and 5

We now consider case 2, in which the coalitions are $\{a_1, a_2\}$ and $\{a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_2\}$, strategy $s_{12,1}$ is to produce $g_1 = 31, g_2 = 0$, expecting profit of 197.16, strategy $s_{12,2}$ is to produce $g_1 = 16, g_2 = 15$ expecting profit of 181.86, strategy $s_{12,3}$ is to produce $g_1 = 0, g_2 = 65$ expecting profit of 240.5. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 43, g_2 = 2$ expecting profit of 262.76, strategy $s_{3,2}$ is to produce $g_1 = 22, g_2 = 21$ expecting profit of 244.02, strategy $s_{3,3}$ is to produce $g_1 = 11, g_2 = 74$ expecting profit of 339.24. We plot agents' strategies and respective outcomes in table 5.21.

Table. 5.21. Strategies of $\{a_1, a_2\}$, $\{a_3\}$ agent

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1, a_2	$s_{12,1}$	143.84, 199.52	169.88, 120.56	183.52, 65.12
	$s_{12,2}$	83.84, 225.32	97.28, 133.76	104.32, 71.72
	$s_{12,3}$	0, 252.84	0, 147.84	0, 78.76

Following the same analytic process, the outcome of case 2 is $\{a_1, a_2\}$ play $s_{12,1}$ and gain 143.84, and $\{a_3\}$ plays $s_{3,1}$ and gains 199.52.

We now consider case 3, in which the coalitions are $\{a_1, a_3\}$ and $\{a_2\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_3\}$, strategy $s_{13,1}$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 280, strategy $s_{13,2}$ is to produce $g_1 = 25, g_2 = 24$ expecting profit of 273.48, strategy $s_{13,3}$ is to produce $g_1 = 16, g_2 = 84$ expecting profit of 390.24. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 13, g_2 = 12$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 50$ expecting profit of 200. We plot agents' strategies and respective outcomes in table 5.22.

Table. 5.22. Strategies of $\{a_1, a_3\}$, $\{a_2\}$ agent

		a_2		
		$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
a_1, a_3	$s_{13,1}$	230, 115	254, 66.04	280, 0
	$s_{13,2}$	140, 140	152, 79.04	165, 0
	$s_{13,3}$	95.36, 149	103.04, 83.72	111.36, 0

Following the same analytic process, the outcome of case 3 is $\{a_1, a_3\}$ play $s_{13,1}$ and gain 230, and $\{a_2\}$ plays $s_{2,1}$ and gains 115

We now consider case 4, in which the coalitions are $\{a_1\}$ and $\{a_2, a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1\}$, strategy $s_{1,1}$ is to produce $g_1 = 6, g_2 = 0$, expecting profit of 44.16, strategy $s_{1,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{1,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. For a_2, a_3, strategy $s_{23,1}$ is to produce $g_1 = 68, g_2 = 1$ expecting profit of 336.82, strategy $s_{23,2}$ is to produce $g_1 = 36, g_2 = 35$ expecting profit of 372.26, strategy $s_{23,3}$ is to produce $g_1 = 35, g_2 = 90$ expecting profit of 505. We plot agents' strategies and respective outcomes in table 5.23.

Table. 5.23. Strategies of $\{a_1\}$, $\{a_2, a_3\}$ agent

		a_2, a_3		
		$s_{23,1}$	$s_{23,2}$	$s_{23,3}$
a_1	$s_{1,1}$	27.84, 315.52	35.52, 213.12	35.76, 208.6
	$s_{1,2}$	14.28, 323.68	18.12, 217.44	18.24, 212.8
	$s_{1,3}$	0, 331.84	0, 221.76	0, 217

Following the same analytic process, the outcome of case 4 is $\{a_1\}$ play $s_{1,1}$ and gain 27.84, and $\{a_2, a_3\}$ plays $s_{23,1}$ and gains 315.52

5.5.5 Trend 5

Case 1

For a_1, strategy $s_{1,1}$ is to produce $g_1 = 43, g_2 = 2$, expecting profit of 262.76, strategy $s_{1,2}$ is to produce $g_1 = 22, g_2 = 21$ expecting profit of 244.02, strategy $s_{1,3}$ is to produce $g_1 = 11, g_2 = 74$ expecting profit of 339.24. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 13, g_2 = 12$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 50$ expecting profit of 200. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 6, g_2 = 0$ expecting profit of 44.16, strategy $s_{3,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. Unfortunately, the above expectations are not the final outcomes due to the fact that the actual prices depends on the number of goods being produced into the market. The higher the number, the lower the price.

Table. 5.24. Strategies of $\{a_1\}$, $\{a_2\}$, $\{a_3\}$ agent

$s_{1,1}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	199.52, 116, 27.84	204.68, 119, 14.28	209.84, 122, 0
	$s_{2,2}$	220.16, 66.56, 30.72	225.32, 68.12, 15.72	230.48, 69.68, 0
	$s_{2,3}$	242.52, 0,33.84	247.68, 0, 17.28	252.84, 0, 0

$s_{1,2}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	120.56, 137, 32.88	123.2, 140, 16.8	125.84, 143, 0
	$s_{2,2}$	131.12,77.48, 35.76	133.76, 79.04, 18.24	136.4, 80.6, 0
	$s_{2,3}$	142.56, 0, 38.88	145.2, 0, 19.8	147.84, 0, 0

$s_{1,3}$

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_2	$s_{2,1}$	65.12, 148, 35.52	66.44, 151, 18.12	67.76, 154, 0
	$s_{2,2}$	70.4, 83.2, 38.4	71.72, 84.76, 19.56	73.04, 86.32, 0
	$s_{2,3}$	76.12, 0, 41.52	77.44, 0, 21.12	78.76, 0, 0

We plot agents' strategies and respective outcomes in table 5.24. The first outcome

(199.52, 116, 27.84) means agent a_1 gains 199.52 by producing 43 units of g_1, agent a_2 gains 116 by producing 25 units of g_1 and agent a_3 gains 27.84 by producing 6 units of g_1. This is because the total number of g_1 is 74 lowering the unit price to 4.64. We now focus on agent a_3. Suppose, agent a_1 plays $s_{1,1}$. If agent a_2 plays $s_{2,1}$, the best strategy for a_3 is $s_{3,1}$ because it gives the maximal payoff 27.84 to a_3. This is also the case even a_2 chose to play $s_{2,2}$ or $s_{2,3}$. This strategy $s_{1,3}$ is also the best choice for a_3 if a_1 plays $s_{1,2}$, regardless of what strategy a_2 plays, or $s_{1,3}$, regardless of what strategy a_2 plays. We can conclude that in case 1), where agents do not cooperate, agent a_3 will always play $s_{1,3}$. Following the same analytic process, the outcome of case 1) is a_1 plays $s_{1,1}$ and gains 199.52, a_2 plays $s_{2,1}$ and gains 116, and a_3 plays $s_{3,1}$ and gains 27.84.

Case 2, 3, 4 and 5

We now consider case 2, in which the coalitions are $\{a_1, a_2\}$ and $\{a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_2\}$, strategy $s_{12,1}$ is to produce $g_1 = 68, g_2 = 1$, expecting profit of 336.82, strategy $s_{12,2}$ is to produce $g_1 = 39, g_2 = 38$ expecting profit of 396.68, strategy $s_{12,3}$ is to produce $g_1 = 38, g_2 = 82$ expecting profit of 506.56. For a_3, strategy $s_{3,1}$ is to produce $g_1 = 6, g_2 = 0$ expecting profit of 44.16, strategy $s_{3,2}$ is to produce $g_1 = 3, g_2 = 2$ expecting profit of 32.36, strategy $s_{3,3}$ is to produce $g_1 = 0, g_2 = 15$ expecting profit of 70.5. We plot agents' strategies and respective outcomes in table 5.25.

Table. 5.25. Strategies of $\{a_1, a_2\}$, $\{a_3\}$ agent

		a_3		
		$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1, a_2	$s_{12,1}$	315.52, 27.84	323.68, 14.28	331.84, 0
	$s_{12,2}$	226.2, 34.8	230.88, 17.76	235.56, 0
	$s_{12,3}$	221.92, 35.04	226.48, 17.88	231.04, 0

Following the same analytic process, the outcome of case 2 is $\{a_1, a_2\}$ play $s_{12,1}$ and gain 315.52, and $\{a_3\}$ plays $s_{3,1}$ and gains 27.84.

We now consider case 3, in which the coalitions are $\{a_1, a_3\}$ and $\{a_2\}$. Among many possibilities, agents may have the following plans. For $\{a_1, a_3\}$, strategy $s_{13,1}$ is to produce $g_1 = 50, g_2 = 0$, expecting profit of 280, strategy $s_{13,2}$ is to produce $g_1 = 25, g_2 = 24$ expecting profit of 273.48, strategy $s_{13,3}$ is to produce $g_1 = 16, g_2 = 84$ expecting profit of 390.24. For a_2, strategy $s_{2,1}$ is to produce $g_1 = 25, g_2 = 0$ expecting profit of 165, strategy $s_{2,2}$ is to produce $g_1 = 13, g_2 = 12$ expecting profit of 149.16, strategy $s_{2,3}$ is to produce $g_1 = 0, g_2 = 50$ expecting profit of 200. We plot agents' strategies and respective outcomes in table 5.26.

Table. 5.26. Strategies of $\{a_1, a_3\}$, $\{a_2\}$ agent

		a_2		
		$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
a_1, a_3	$s_{13,1}$	230, 115	254, 66.04	280, 0
	$s_{13,2}$	140, 140	152, 79.04	165, 0
	$s_{13,3}$	95.36, 149	103.04, 83.72	111.36, 0

Following the same analytic process, the outcome of case 3 is $\{a_1, a_3\}$ play $s_{13,1}$ and gain 230, and $\{a_2\}$ plays $s_{2,1}$ and gains 115

We now consider case 4, in which the coalitions are $\{a_1\}$ and $\{a_2, a_3\}$. Among many possibilities, agents may have the following plans. For $\{a_1\}$, strategy $s_{1,1}$ is to produce $g_1 = 43, g_2 = 2$, expecting profit of 262.76, strategy $s_{1,2}$ is to produce $g_1 = 22, g_2 = 21$ expecting profit of 244.02, strategy $s_{1,3}$ is to produce $g_1 = 11, g_2 = 74$ expecting profit of 339.24. For a_2, a_3, strategy $s_{23,1}$ is to produce $g_1 = 31, g_2 = 0$ expecting profit of 197.16, strategy $s_{23,2}$ is to produce $g_1 = 16, g_2 = 15$ expecting profit of 181.86, strategy $s_{23,3}$ is to produce $g_1 = 0, g_2 = 65$ expecting profit of 240.5. We plot agents' strategies and respective outcomes in table 5.27.

Table. 5.27. Strategies of $\{a_1\}$, $\{a_2, a_3\}$ agent

		a_2, a_3		
		$s_{23,1}$	$s_{23,2}$	$s_{23,3}$
a_1	$s_{1,1}$	199.52, 143.84	225.32, 83.84	252.84, 0
	$s_{1,2}$	120.56, 169.88	133.76, 97.28	147.84, 0
	$s_{1,3}$	65.12, 183.52	71.72, 104.32	78.76, 0

Following the same analytic process, the outcome of case 4 is $\{a_1\}$ play $s_{1,1}$ and gain 199.52, and $\{a_2, a_3\}$ plays $s_{23,1}$ and gains 143.84

Compensate

By following the complex analytic processes, we receive the outcomes of each trend as shown in table 5.7. The outstanding figure is the grand coalition's value remain the same and is the best strategy for all agents. Another notable result is that a_2's coalition values change all the times even it receives the same number of resources in all trends. Lastly, trend 2 and 3 are diagonally similar as well as trend 4 and 5.

Table. 5.28. Payoff of agent in all 5 trends

Coalition Structure	Trend 1	Trend 2	Trend 3	Trend 4	Trend 5
a_1	115	86.4	34.56	27.84	199.52
a_2	115	144	144	116	116
a_3	115	34.56	86.4	199.52	27.84
a_1, a_2	230	309.4	178.56	143.84	315.52
a_1, a_3	230	230	230	230	230
a_2, a_3	230	178.56	309.4	315.52	143.84
a_1, a_2, a_3	533.76	533.76	533.76	533.76	533.76
Payoff by SV	177.92,177.92,177.92	116.0,210.42,207.34	207.34,210.42,116	263.43,178.59,91.75	91.75,178.59,263.43

The agents' payoffs are shown in table 5.28. As one may expect, the agents' payoffs have some pattern reflecting the patterns of resources the are allocated. In trend 2 and 3, the payoffs of a_1 and a_3 are diagonally similar, i.e. 116 and 207.34, while a_2 receives 210.42. In trend 4 and 5, the payoffs of a_1 and a_3 are also diagonally similar, i.e. 263.43 and 91.75, while a_2 receives 178.59.

5.5.6 For example to find nash equilibrium

Form Strategy Profile and Payoff Vector with Strategies and Outcomes of Trend 1 Case 1, as shown in table 5.8. That the grand coalition's value is the same for all trends, it is the best strategy for all agents. The coalition value of a_2 in all trends vary despite the same number of resources in all trends because of number of goods produced by other agents. Also, there is a diagonal similarity between Trend 2, Trend 3 and Trend 4, Trend 5. Based on the given resources in each game, there can be many possible plans for producing goods for each agent. Each of these plans can be considered a strategy of each agent in each game. Since there are so many possibilities, we consider only three strategies for each agent in each game. The first strategy is to produce only g_1. The second strategy is to optimally produce both g_1 and g_2. The third strategy is to produce only g_2. Hence, there are strategies $\{s_{1,1}, s_{1,2}, s_{1,3}\}$, $\{s_{2,1}, s_{2,2}, s_{2,3}\}$ and $\{s_{3,1}, s_{3,2}, s_{3,3}\}$ for agent a_1, a_2 and a_3, respectively. For each strategic profile, the optimal plan for each agent will be computed and the actual payoff will also be calculated taken into account the total amount of goods and respective unit prices. Note that the actual payoff for each agent may be less than its expected value. Given a strategic profile, the payoffs for all agents a *payoff vector* (v_1, v_2, v_3)

Following SFG structure, there are twenty-seven strategic profiles for each game and their respective payoff vectors will be presented in the game table accordingly. Strategic profile $(s_{1,1}, s_{2,1}, s_{3,1})$, indicating that agent a_1 plays $s_{1,1}$, a_1 plays $s_{1,1}$, a_1 plays $s_{1,1}$, is associated with payoff vector (v_1, v_2, v_3), indicating that payoffs for agent a_1, a_2 and a_3 are v_1, v_2 and v_3, respectively, as shown in table 5.29.

Table. 5.29. Strategy Profile and Payoff Vector

Strategy Profile			**Payoff Vector**		
a_1	a_2	a_3	v_1	v_2	v_3
$s_{1,1}$	$s_{2,1}$	$s_{3,1}$	115	115	115
$s_{1,1}$	$s_{2,1}$	$s_{3,2}$	122	122	87.84
$s_{1,1}$	$s_{2,1}$	$s_{3,3}$	140	140	0
$s_{1,1}$	$s_{2,2}$	$s_{3,1}$	122	87.84	122
$s_{1,1}$	$s_{2,2}$	$s_{3,2}$	129	92.88	92.88
$s_{1,1}$	$s_{2,2}$	$s_{3,3}$	147	105.84	0
$s_{1,1}$	$s_{2,3}$	$s_{3,1}$	140	0	140
$s_{1,1}$	$s_{2,3}$	$s_{3,2}$	147	0	105.84
$s_{1,1}$	$s_{2,3}$	$s_{3,3}$	165	0	0
$s_{1,2}$	$s_{2,1}$	$s_{3,1}$	87.84	122	122
$s_{1,2}$	$s_{2,1}$	$s_{3,2}$	92.88	129	92.88
$s_{1,2}$	$s_{2,1}$	$s_{3,3}$	113.04	157	0
$s_{1,2}$	$s_{2,2}$	$s_{3,1}$	92.88	92.88	129
$s_{1,2}$	$s_{2,2}$	$s_{3,2}$	97.92	97.92	97.92
$s_{1,2}$	$s_{2,2}$	$s_{3,3}$	110.88	110.88	0
$s_{1,2}$	$s_{2,3}$	$s_{3,1}$	105.84	0	147
$s_{1,2}$	$s_{2,3}$	$s_{3,2}$	110.88	0	110.88
$s_{1,2}$	$s_{2,3}$	$s_{3,3}$	123.84	0	0
$s_{1,3}$	$s_{2,1}$	$s_{3,1}$	0	140	140
$s_{1,3}$	$s_{2,1}$	$s_{3,2}$	0	147	105.84
$s_{1,3}$	$s_{2,1}$	$s_{3,3}$	0	165	0
$s_{1,3}$	$s_{2,2}$	$s_{3,1}$	0	105.84	147
$s_{1,3}$	$s_{2,2}$	$s_{3,2}$	0	110.88	110.88
$s_{1,3}$	$s_{2,2}$	$s_{3,3}$	0	123.84	0
$s_{1,3}$	$s_{2,3}$	$s_{3,1}$	0	0	165
$s_{1,3}$	$s_{2,3}$	$s_{3,2}$	0	0	123.84
$s_{1,3}$	$s_{2,3}$	$s_{3,3}$	0	0	0

5.5.7 Discussion

Given resources in Trend 1, we deliberately compute for the best plan for each agent as a sole seller and compute for agents' payoff in strategic form game. Table 5.29 shows both agents' expected profits as sole sellers in the market and agents' payoffs as players in game of Trend 1. As a sole seller in the market, a_1, a_2, a_3 expect to produce 18 units of g_1, 17 units of g_2 and receive profit of 203.6. Since they are all in the market, we have to carefully consider the outcome of the game. Let's consider agent a_3, Assuming, agent a_1 plays $s_{1,1}$ and agent a_2 plays $s_{2,1}$, agent a_3's best strategy is $s_{3,1}$, receiving the highest payoff 115. This remains the same when agent a_2 plays $s_{2,2}$ or $s_{2,3}$. If a_1 plays $s_{1,2}$, strategy $s_{1,3}$ remains the best choice for a_3, no matters what a_2 plays. In other words, agent a_3 always plays $s_{1,3}$. By similarly analyzing the situation, the strategic profile $(s_{1,1}, s_{2,1}, s_{3,1})$ is the outcome and is also in Nash Equilibrium. Their actual payoffs drop to 115 each.

5.6 Conclusion

We study non-cooperative bakery game. A wide range of amount of resources into 5 trends. Trend 1 is used as a reference. Trend 2, 3 and Trend 4, 5 are diagonally similar. Given certain technology matrix and price functions, we find that within our settings agents' strategies remain unchanged even though resources vary upto 75%. Furthermore, agents' payoffs changes relatively small. In the future, this research can be extended to consider more complex situations with more details. While there are a small number of agents and actions are used in this research, there should be more agents and actions involved. Furthermore, there could be algorithms working on other aspects, including efficiency, etc.

CHAPTER 6

FAIR PAYOFF IN NON-COOPERATIVE GAME

Nash proves in this thesis [35] that there are always equilibria in n-person strategic form game. However, it was later proves that it is not actually the case in these works, particular game are invented and considered of a small number of agents, each of which having a couple of strategies.

In real world domains, number of agents can be up to hundreds or thousands, resulting in much larger search space. It is important that we know how we can find satisfactory results in timely section. Knowing such trends we can possibly predict the outcome of the game given relation of agents payoffs. more interesting, the acquired knowledge may lead to the design of rules of game (or set of strategic profiles) in order to achieve desired results.

However, a game of a small number of agents can still be challenging because search apace of possible outcomes, deriving form a small number of possible strategy payoffs, can still be very large. In typical research in game theory, a game consisting of a couple of agents, each of which possesses a couple of strategies, will be consider. Here, we also start off with a couple of agents, but we consider a larger search space by including all possible combination of agents strategies. We expressively explore 1.8 hundred thousand millions of game to find out patterns or trend of games with and without equilibrium.

6.1 Introduction to strategy in normal form game

In gaem theory, the first game form is extensive form game, a non-cooperative game, where sequence of actions are associated with payoffs, in form of tree, until leaves of the tree. Here, we present the definition of strategic form game, defined by [33], for the sake of completeness. It is described that the game is defined by exhibiting on each side of the matrix the different players (here players 1 and 2), each strategy or choice they can make (here strategies A and B) and sets of payoffs they will each receive for a given strategy ($p_{1A}, p_{2A}; p_{1A}, p_{2B}; p_{1B}, p_{2A}; p_{1B}, p_{2B}$). It is explained [33] that the strategic form allows us to quickly analyse each possible outcome of a game. In the depicted matrix, if player 1 chooses strategy A and player 2 chooses strategy B, the set of payoffs given by the outcome would be p_{1A}, p_{2B}. If player 1 chooses strategy B and player 2 chooses strategy A, the set of payoffs would be p_{1B}, p_{2A}.

Table. 6.1. strategies A and B

		PLAYER 2	
		Strategy A	**Strategy B**
PLAYER 2	**Strategy A**	p_{1A}, p_{2A}	p_{1A}, p_{2A}
	Strategy B	p_{1B}, p_{2A}	p_{1B}, p_{2B}

Gallego [33] suggests that an n-person game in strategic form (or, normal form) has 3 essential elements

1. A finite set of players $I = 1, 2, ..., n$.

2. For each player i, a finite set of strategies S_i. Let $s = (s_1, s_2, ...s_n)$ denote an n-tuple of strategies, one for each player. This n-tuple is called a strategy combination or strategy profile. The set $S = S_1 \times S_2 \times ... \times S_n$ denotes the set of n-tuple of strategies.

3. For each player i, there is a payoff function $P_i : S \to R$, which associates with each strategy combination $(s1, s2, ..., sn)$, a payoff $P_i(s_1, s_2, ..., s_n)$ for player i. Since we have one such function for each player i, in all we have n such functions.

6.2 NE in strategy in normal form game

The most famous, widely adopted and studied solution concept of strategic form game is Nash Equilibrium [35]. There are so many re-invented definitions, in addition to the original one [35]. Here we use the definition defined by [38]: "A Nash equilibrium [38] is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^*." It is explained [38] that A Nash equilibrium corresponds to a steady state. If, when ever the game is played, the action profile is the same Nash equilibrium a^*, then no player has a reason to choose any action different from her component of a^*; there is no pressure on the action profile to change. It is believed that [38] Nash equilibrium embodies a stable “social norm”: if everyone else adheres to it, no individual wishes to deviate from it. Furthermore, it is stated [38] that the players’ beliefs about each other’s actions are correct implies, in particular, that two players’ beliefs about a third player’s action are the same. For this reason, the condition is sometimes said to be that the players’ “expectations are coordinated”.

Here, we present a definition of a Nash equilibrium, presented by [38]. Let a be an action profile, in which the action of each player i is a_i. Let a'_i be any action of player i (either equal to a_i, or different from it). Then (a'_i, a_{-i}) denotes j except i chooses her action a_j as specified by a, whereas player i chooses a'_i. (The $-i$ subscript on a stands for "except i".) That is, (a'_i, a_{-i}) is the action profile in which all the players other than i adhere to a while i “deviates” to a'_i. (If $a'_i = a_i$ then of course $(a'_i, a_{-i} = (a_i, a_{-i}) = a$.) If there are three players, for example, then (a'_2, a_{-2}) is the action profile in which players 1 and 3 adhere to a (player 1 chooses a_1, player 3 chooses a_3) and player 2 deviates to a'_2. Using this notation, we can restate the condition for an action profile a^* to be a Nash equilibrium: no player i has any action a_i for which she prefers (a_i, a^*_{-i}) to a^*. Equivalently, for every player i and every action ai of player i, the action profile a^* is at least as good for player i as the action profile (a_i, a^*_{-i}).

6.3 Strategy profile spectrum

We consider games of three agents, each of which has true strategies. In this set of games. The payoffs for agents range from one to three. That is the payoff vectors are 27 combinations of there value, e.g. (0, 0, 0), (0, 0, 1), (0, 1, 0), ..., (2, 2, 2). Having three agents with these payoff combinations, there can be as many as $27 \times 27 \times 27 \times 27 \times 27 \times 27 \times 27 \times 27 = 282,429,536,481$ games. shown in table 6.2.

Table. 6.2. Payoff Combinations

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0

Game 1

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 1

Game 2

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 2

Game 3

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	0, 1, 0

Game 4

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	1, 1, 0

.
.
.

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	1, 1, 1

.
.
.

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	1, 1, 2

.
.
.

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	1, 2, 0

.
.
.

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	1, 2, 1

.
.
.

0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
0, 0, 0	0, 0, 0	0, 0, 0	1, 2, 2

Game 282,429,536,479

2, 2, 2	2, 2, 2	2, 2, 2	2, 2, 2
2, 2, 2	2, 2, 2	2, 2, 2	2, 1, 3

.
.

Game 282,429,536,480

2, 2, 2	2, 2, 2	2, 2, 2	2, 2, 2
2, 2, 2	2, 2, 2	2, 2, 2	2, 2, 2

.
.

Game 282,429,536,481

Table 6.2 is an overview of payoff combinations of all games from 1 - 282,429,536,481 games, the components of Game 1 are 3 agents, each of which has 2 strategy. With these payoff vector, there can be as many as $3 \times 3 \times 3 = 27$ payoff vectors. By strategy combination in which each agent. Agent a, there are 2 strategy combination, a_1 and a_2. Agent b, there are 2 strategy combination, b_1 and b_2. And agent C, there are 2 strategy combination, c_1 and c_2. As shown in table 6.3

Table. 6.3. Strategy combination

a_1 **C**

		C_1	C_2
b	b_1	0, 0, 0	0, 0, 0
	b_2	0, 0, 0	0, 0, 0

a_2 **C**

		C_1	C_2
b	b_1	0, 0, 0	0, 0, 0
	b_2	0, 0, 0	0, 0, 0

Game1

Addition to we consider each game to search NE. By division game that start P_1 of first payoff combinations from 0 - 2 payoff combinations than we search NE in every payoff combinations for $P_1, P_2, ..., P_8$ of all 282,429,536,481 games

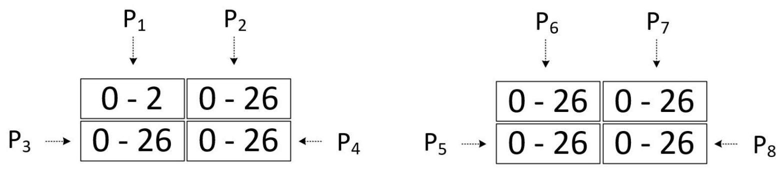


Figure. 6.1. Example payoff combinations.

6.4 External Relational Characteristic

There are external relationship of these games. The complete ranges are shown in table 6.4 and 6.5. In table 6.4, the integer portion of those setting is shown. In table 6.5 the number of games is shown in form of a binomial coefficient.

Table. 6.4. Integer partition

Case																										
Z	**Y**	**X**	**W**	**V**	**U**	**T**	**S**	**R**	**Q**	**P**	**O**	**N**	**M**	**L**	**K**	**J**	**I**	**H**	**G**	**F**	**E**	**D**	**C**	**B**	**A**	**0**
26	**25**	**24**	**23**	**22**	**21**	**20**	**19**	**18**	**17**	**16**	**15**	**14**	**13**	**12**	**11**	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**0**
8	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
7	0	1	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
7	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	1
6	2	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
6	0	2	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
6	0	0	2	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	2	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	2	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	2	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	2	0
6	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	2
5	3	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	3	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	3	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	3	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	3	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	3	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	3	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	3	0
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	3
5	2	1	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	2	1	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	2	1	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	2	1	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	2	1	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	2	1	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	2	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	2	1
5	1	1	1	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	1	1	1	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	1	1	1	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	1	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	1	1	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	1	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	6	2
0	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	7	1
0	0	0	0	0	0	0	0	0	0	0	0	0	···	···	···	0	0	0	0	0	0	0	0	0	0	8

Table. 6.5. Number of games

	8	**7**	**6**	**5**	**4**	**3**	**2**	**1**
Z	${}^{8}C_8$	${}^{26}C_1$	${}^{26}C_1$ + ${}^{26}C_2$	${}^{26}C_1$ + ${}^{26}C_2$ + ${}^{26}C_3$	${}^{26}C_1$ + ${}^{26}C_2$ + ${}^{26}C_3$ + ${}^{26}C_4$	${}^{26}C_1$ + ${}^{26}C_2$ + ${}^{26}C_3$ + ${}^{26}C_4$ + ${}^{26}C_5$	${}^{26}C_1$ + ${}^{26}C_2$ + ${}^{26}C_3$ + ${}^{26}C_4$ + ${}^{26}C_5$ + ${}^{26}C_6$	${}^{26}C_1$ + ${}^{26}C_2$ + ${}^{26}C_3$ + ${}^{26}C_4$ + ${}^{26}C_5$ + ${}^{26}C_6$ + ${}^{26}C_7$
Y	${}^{8}C_8$	${}^{25}C_1$	${}^{25}C_1$ + ${}^{25}C_2$	${}^{25}C_1$ + ${}^{25}C_2$ + ${}^{25}C_3$	${}^{25}C_1$ + ${}^{25}C_2$ + ${}^{25}C_3$ + ${}^{25}C_4$	${}^{25}C_1$ + ${}^{25}C_2$ + ${}^{25}C_3$ + ${}^{25}C_4$ + ${}^{25}C_5$	${}^{25}C_1$ + ${}^{25}C_2$ + ${}^{25}C_3$ + ${}^{25}C_4$ + ${}^{25}C_5$ + ${}^{25}C_6$	${}^{25}C_1$ + ${}^{25}C_2$ + ${}^{25}C_3$ + ${}^{25}C_4$ + ${}^{25}C_5$ + ${}^{25}C_6$ + ${}^{25}C_7$
X	${}^{8}C_8$	${}^{24}C_1$	${}^{24}C_1$ + ${}^{24}C_2$	${}^{24}C_1$ + ${}^{24}C_2$ + ${}^{24}C_3$	${}^{24}C_1$ + ${}^{24}C_2$ + ${}^{24}C_3$ + ${}^{24}C_4$	${}^{24}C_1$ + ${}^{24}C_2$ + ${}^{24}C_3$ + ${}^{24}C_4$ + ${}^{24}C_5$	${}^{24}C_1$ + ${}^{24}C_2$ + ${}^{24}C_3$ + ${}^{24}C_4$ + ${}^{24}C_5$ + ${}^{24}C_6$	${}^{24}C_1$ + ${}^{24}C_2$ + ${}^{24}C_3$ + ${}^{24}C_4$ + ${}^{24}C_5$ + ${}^{24}C_6$ + ${}^{24}C_7$
W	${}^{8}C_8$	${}^{23}C_1$	${}^{23}C_1$ + ${}^{23}C_2$	${}^{23}C_1$ + ${}^{23}C_2$ + ${}^{23}C_3$	${}^{23}C_1$ + ${}^{23}C_2$ + ${}^{23}C_3$ + ${}^{23}C_4$	${}^{23}C_1$ + ${}^{23}C_2$ + ${}^{23}C_3$ + ${}^{23}C_4$ + ${}^{23}C_5$	${}^{23}C_1$ + ${}^{23}C_2$ + ${}^{23}C_3$ + ${}^{23}C_4$ + ${}^{23}C_5$ + ${}^{23}C_6$	${}^{23}C_1$ + ${}^{23}C_2$ + ${}^{23}C_3$ + ${}^{23}C_4$ + ${}^{23}C_5$ + ${}^{23}C_6$ + ${}^{23}C_7$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A	${}^{8}C_8$	${}^{1}C_1$	${}^{1}C_1$ + ${}^{1}C_2$	${}^{1}C_1$ + ${}^{1}C_2$ + ${}^{1}C_3$	${}^{1}C_1$ + ${}^{1}C_2$ + ${}^{1}C_3$ + ${}^{1}C_4$	${}^{1}C_1$ + ${}^{1}C_2$ + ${}^{1}C_3$ + ${}^{1}C_4$ + ${}^{1}C_5$	${}^{1}C_1$ + ${}^{1}C_2$ + ${}^{1}C_3$ + ${}^{1}C_4$ + ${}^{1}C_5$ + ${}^{1}C_6$	${}^{1}C_1$ + ${}^{1}C_2$ + ${}^{1}C_3$ + ${}^{1}C_4$ + ${}^{1}C_5$ + ${}^{1}C_6$ + ${}^{1}C_7$
0	${}^{8}C_8$	${}^{0}C_1$	${}^{0}C_1$ + ${}^{0}C_2$	${}^{0}C_1$ + ${}^{0}C_2$ + ${}^{0}C_3$	${}^{0}C_1$ + ${}^{0}C_2$ + ${}^{0}C_3$ + ${}^{0}C_4$	${}^{0}C_1$ + ${}^{0}C_2$ + ${}^{0}C_3$ + ${}^{0}C_4$ + ${}^{0}C_5$	${}^{0}C_1$ + ${}^{0}C_2$ + ${}^{0}C_3$ + ${}^{0}C_4$ + ${}^{0}C_5$ + ${}^{0}C_6$	${}^{0}C_1$ + ${}^{0}C_2$ + ${}^{0}C_3$ + ${}^{0}C_4$ + ${}^{0}C_5$ + ${}^{0}C_6$ + ${}^{0}C_7$

6.5 Algorithm analytic

6.5.1 Integer Partition

Let m be a positive integer. A partition of m is a representation of m as a sum of positive integers, say $m = a_1 + ... + a_n$ The summands $a_1, ..., a_n$ are called the parts of the partition, and their order is ignored. The notation $P(m)$ is used to denote the number of partitions of $m; P(m)$ is called a partition number.

The first few partition numbers are $P(1) = 1, P(2) = 2, P(3) = 3, P(4) = 5, P(5) = 7, P(6) = 11$. As an example, the 11 different partitions of the integer 6

Table. 6.6. Groups partitions

1	6
2	5 + 1
2	4 + 2
3	4 + 1 + 1
2	3 + 3
3	3 + 2 + 1
4	3 + 1 + 1 + 1
3	2 + 2 + 2
4	2 + 2 + 1 + 1
5	2 + 1 + 1 + 1 + 1
6	1 + 1 + 1 + 1 + 1 + 1

Although partitions have been studied by mathematicians for hundreds of years and many interesting results are known, there is no known formula for the values $P(m)$. The growth rate of $P(m)$ is known however; it can be shown the $P(m)$ is $\Theta(e^{\pi\sqrt{2m/3}}/m)$.

A partition $m = a_1 + ... + a_n$ is said to be in standard form if $a_1 \geq a_2 \geq \ldots \geq a_n$. (Note that the 11 partitions of 6 given above are all in standard form.) A partition in standard form as a list. i.e., $[a_1, a_2, \ldots, a_n]$, particularly in Algorithm 2.

Algorithm 2 Integer Partition

```
procedure
    System Initialization
    Read the value
    Set Array a
    Set m
    Set vector partition
    input m, b, n
    if m = 0 then
        a[n] = 0
        r_j = 1
    else
        for each i = 1, i ⩽ min(b, m) do
            a[n + 1] = i
            Set m − i, i, n + 1
          end for
    end if
    for each i = m, i ⩽ 1, i − − do
        a[1] = i
        Set m − i, i, n + 1
    return partition
     end for
    Set n = noa
    Set parts = 2
    for i = m, i < 1, i + + do
        count = 0
        for j = 1, j < m.length, j + + do
            if temp[j] > 0 then
                ++count
             end if
          end for
     end for
```

6.5.2 Subset

A set A is a subset of a set B if all elements of A are also elements of T; B is then a superset of A. It is possible for A and B to be equal; if they are unequal, then A is a proper subset of B. The relationship of one set being a subset of another is called inclusion (or sometimes containment). A is a subset of B may also be expressed as B includes (or contains) A or A is included (or contained) in B.

The subset relation defines a partial order on sets. In fact, the subsets of a given set form a Boolean algebra under the subset relation, in which the join and meet are given by intersection and union, and the subset relation itself is the Boolean inclusion relation. Algorithm 3.

Algorithm 3 Subset

procedure
  System Initialization
  for each i from 1 to N **do**
    let T be a list consisting of $x_i + y$, for all y in S
    let U be the union of T and S
    sort U
    make S empty
    let y be the smallest element of U
    add y to S
    for each element z of U in increasing order **do**
    ▷ Trim the list by eliminating numbers close to one another and throw out elements greater than s.
      if $y + cs/N < z \leqslant s$ **then**
        $y = z$
        add z to S
      end if
    end for
  end for

6.5.3 NE of 3 agents and 2 Strategy Algorithm

In order to fine relationship between agents payoffs profile and predict the outcome of the game, we expressively explore all the possible 27^2 = (0, 0, 0), (0, 0, 1), (0, 0, 2), (0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2), (2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 2, 0), (2, 2, 1) and (2, 2, 2) games. there are 8 levels of loops, each of which represent a combination of agent payoffs in a payoff vector. The value 27 in each loop represent the agent payoffs in one value. For example value

1 represents vector 0, 0, 0), (0, 0, 1), (0, 0, 2), (0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2), 2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 2, 0), (2, 2, 1) and (2, 2, 2)

We find NE, NonNE, and Elapsed to primarily exploring trend of outcomes in a 3-Person-2-Strategy game as shown in Figure 6.2

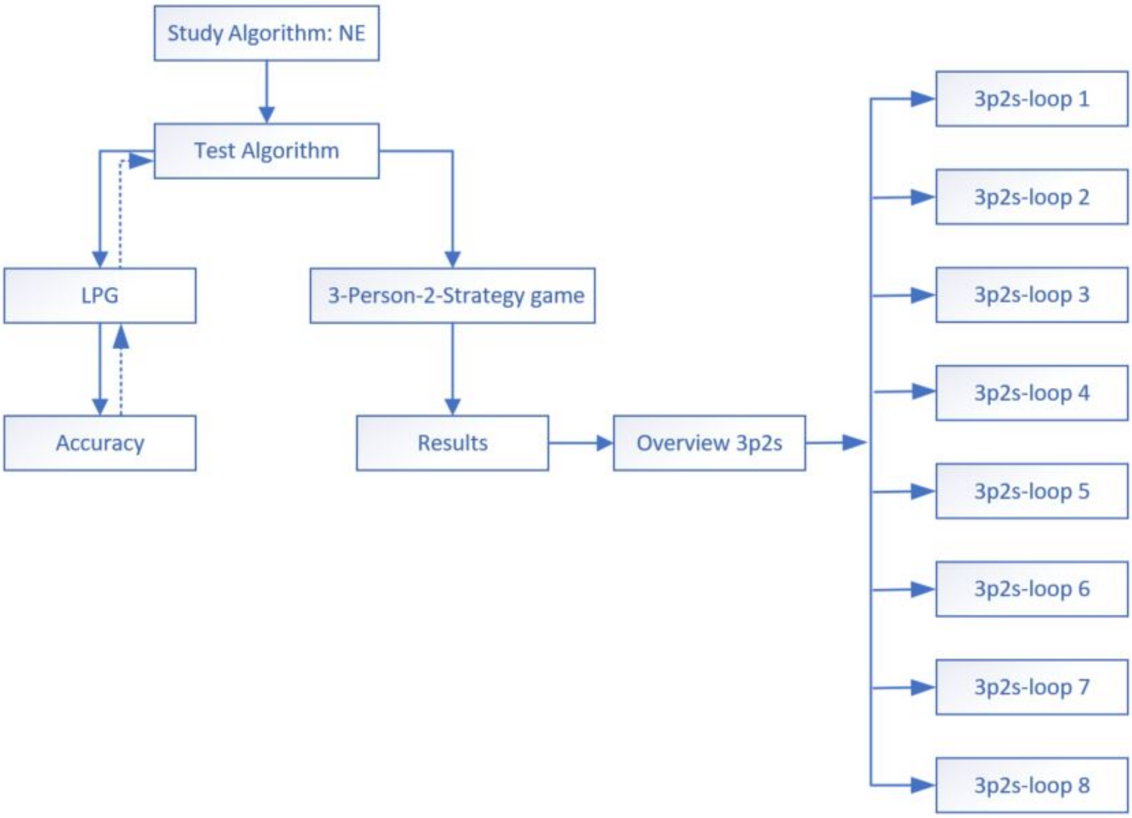


Figure. 6.2. Overview 2p3s.

Algorithm 4 3 agents and 2 Strategy Algorithm

```
procedure
    System Initialization
    Read the value
    Set payoffs = noa
    Set sp = 0
    for each sp < 27, sp++ do
        for each sp < 27, sp1++ do
            for each sp < 27, sp2++ do
                for each sp < 27, sp3++ do
                    for each sp < 27, sp4++ do
                        for each sp < 27, sp5++ do
                            for each sp < 27, sp6++ do
                                for each sp < 27, sp7++ do
                                    for each sp < payoffs, sa++ do                ▷ identify payoff
                                        for each sb < payoffs, sb++ do
                                            for each sc < payoffs, sc++ do
                                                for each a < payoffs, a++ do
                                                    payoffs[sa][sb][sc] = p3x3[sa * 2 * 2 + sb * 2 + sc];
                                                    boolean ne = true
                                                    double p0 = payoffs[i][j][k][a0]
                                                    for each st < payoffs.length, st++ do
                                                        if p0 < payoffs[st][j][k][a0] then
                                                            ne = false
                                                            break
                                                        end if
                                                    end for
                                                    double p1 = payoffs[i][j][k][a1]
                                                    for each st < payoffs[i].length, st++ do
                                                        if p1 < payoffs[st][j][k][a0] then
                                                            ne = false
                                                            break
                                                        end if
                                                    end for
                                                    double p1 = payoffs[i][j][k][a2]
                                                    for each st < payoffs[i].length, st++ do
                                                        if p2 < payoffs[st][j][k][a0] then
                                                            ne = false
                                                            break
                                                        end if
                                                    end for
                                                    if ne then
                                                        Print "profile[i][j][k]:Payoff is IN NE. "
                                                        necount++
                                                    else
                                                        Print "profile[i][j][k]:Payoff is not IN NE. "
                                                        nonnecount++
                                                    end if
                                                end for
                                            end for
                                        end for
                                    end for
                                    Set count++
                                    if count mod mileStone == 0 then
                                        time = System currentTimeMillis
                                        elapsed = time - start
                                        Print count, necounts, nonnecounts, nonnecount, elapsed
                                    end if
                                end for
                            end for
                        end for
                    end for
                end for
            end for
        end for
    end for
```

6.6 Result

6.6.1 NE Result

Table. 6.7. Experiment NE Setting (8 - 5)

Pattern	8		7		6		5	
	#NE	%	#NE	%	#NE	%	#NE	%
0	0	0	0	0	0	0	0	0
A	8	0.0000000	28	0.0000000	56	0.0000000	70	0.0000000
B	1,024	0.0000004	1,792	0.0000006	1,792	0.0000006	1,120	0.0000004
C	17,496	0.0000062	20,412	0.0000072	13,608	0.0000048	5,606	0.0000020
D	130,944	0.0000464	114,592	0.0000406	57,312	0.0000203	17,916	0.0000063
E	625,000	0.0002213	437,500	0.0001549	175,000	0.0000620	43,750	0.0000155
F	2,237,760	0.0007923	1,302,912	0.0004613	432,384	0.0001531	89,696	0.0000318
G	6,577,368	0.0023289	3,289,468	0.0011647	940,296	0.0003329	168,006	0.0000595
H	16,777,216	0.0059403	7,340,032	0.0025989	1,835,008	0.0006497	286,720	0.0001015
I	38,263,752	0.0135481	14,818,140	0.0052467	3,265,272	0.0011561	444,742	0.0001575
J	79,466,096	0.0281366	27,502,948	0.0097380	5,464,024	0.0019347	688,892	0.0002439
K	153,394,536	0.0543125	48,276,124	0.0170932	8,828,904	0.0031261	1,016,870	0.0003600
L	278,634,600	0.0986563	79,560,504	0.0281700	13,238,016	0.0046872	1,414,140	0.0005007
M	489,057,160	0.1731608	132,299,760	0.0468435	20,560,816	0.0072800	1,993,064	0.0007057
N	832,449,912	0.2947461	209,668,444	0.0742374	30,075,648	0.0106489	2,688,620	0.0009520
O	1,318,363,992	0.4667940	302,233,788	0.1070121	40,590,600	0.0143719	3,474,374	0.0012302
P	2,103,484,080	0.7447819	464,031,256	0.1642998	58,443,768	0.0206932	4,583,184	0.0016228
Q	3,282,709,384	1.1623109	675,851,932	0.2392993	79,511,992	0.0281529	5,846,470	0.0020701
R	4,800,622,464	1.6997594	922,285,440	0.3265542	101,198,592	0.0358314	7,047,136	0.0024952
S	6,934,186,248	2.4551916	1,261,154,884	0.4465379	132,853,224	0.0470394	8,918,882	0.0031579
T	9,834,839,808	3.4822278	1,700,725,408	0.6021769	172,413,696	0.0610466	11,043,776	0.0039103
U	13,721,158,296	4.8582590	2,249,197,956	0.7963749	215,533,176	0.0763140	13,231,970	0.0046851
V	19,194,083,904	6.7960611	3,079,703,248	1.0904324	284,100,352	0.1005916	16,326,192	0.0057806
W	26,630,055,288	9.4289201	4,100,737,444	1.4519506	359,277,144	0.1272095	19,579,106	0.0069324
X	34,720,373,088	12.2934639	4,979,854,944	1.7632203	421,547,520	0.1492576	22,719,424	0.0080443
Y	47,433,848,536	16.7949320	6,724,153,492	2.3808252	543,577,624	0.1924649	27,312,194	0.0096704
Z	64,254,481,408	22.7506238	8,649,641,728	3.0625840	665,075,840	0.2354838	31,776,008	0.0112510

Table. 6.8. Experiment NE Setting (4 - 1)

Pattern	4		3		2		1	
	#NE	%	#NE	%	#NE	%	#NE	%
0	0	0	0	0	0	0	1	0.000000000
A	56	0.0000000	28	0.0000000	8	0.000000003	1	0.000000000
B	448	0.0000002	112	0.0000000	16	0.000000006	1	0.000000000
C	1,512	0.0000005	252	0.0000001	24	0.000000008	1	0.000000000
D	3,584	0.0000013	448	0.0000002	32	0.000000011	1	0.000000000
E	7,000	0.0000025	700	0.0000002	40	0.000000014	1	0.000000000
F	12,096	0.0000043	1,008	0.0000004	48	0.000000017	1	0.000000000
G	19,208	0.0000068	1,372	0.0000005	56	0.000000020	1	0.000000000
H	28,672	0.0000102	1,792	0.0000006	64	0.000000023	1	0.000000000
I	40,824	0.0000145	2,268	0.0000008	72	0.000000025	1	0.000000000
J	56,000	0.0000198	2,800	0.0000010	80	0.000000028	1	0.000000000
K	74,536	0.0000264	3,388	0.0000012	88	0.000000031	1	0.000000000
L	96,768	0.0000343	4,032	0.0000014	96	0.000000034	1	0.000000000
M	123,032	0.0000436	4,732	0.0000017	104	0.000000037	1	0.000000000
N	153,664	0.0000544	5,488	0.0000019	112	0.000000040	1	0.000000000
O	189,000	0.0000669	6,300	0.0000022	120	0.000000042	1	0.000000000
P	229,376	0.0000812	7,168	0.0000025	128	0.000000045	1	0.000000000
Q	275,128	0.0000974	8,092	0.0000029	136	0.000000048	1	0.000000000
R	326,592	0.0001156	9,072	0.0000032	144	0.000000051	1	0.000000000
S	384,104	0.0001360	10,108	0.0000036	152	0.000000054	1	0.000000000
T	448,000	0.0001586	11,200	0.0000040	160	0.000000057	1	0.000000000
U	518,616	0.0001836	12,348	0.0000044	168	0.000000059	1	0.000000000
V	596,288	0.0002111	13,552	0.0000048	176	0.000000062	1	0.000000000
W	681,352	0.0002412	14,812	0.0000052	184	0.000000065	1	0.000000000
X	774,144	0.0002741	16,128	0.0000057	192	0.000000068	1	0.000000000
Y	875,000	0.0003098	17,500	0.0000062	200	0.000000071	1	0.000000000
Z	944,271	0.0003343	16,050	0.0000057	119	0.000000042	0	0

6.6.2 Result Primarily Exploring Trend of outcomes in a 3-Person-2-Strategy game

We divided the situation of NE searches into 9 stages, as follows: 0-3, 3-6, 6-9, 9-12, 12-15, 15-18, 18-21, 21-24, 24-27 We want to consider the results. To find the pattern of numbers that derive NE Of Outcome in the game. In this step We want to test the accuracy of the algorithm to find the NE of Primarily Exploring Trend of outcomes in a 3-Person-2-Strategy game.

6.7 Experiment

In each loop we have all a profiles which we can fine the relationship between agents payoff and the outcome of the game. After all we possibly can predict the outcome of the game given trends of relation of agents payoff.

6.7.1 Overview 3p2s

The result of the process of Overview 3p2s. Divided into 5 parts as follows,

1. Results from finding NE Count.
2. Results from finding the difference NE Count.
3. Results from NonNE Count results.
4. Results of finding differences in NonNE Count.
5. Results from finding differences Elapsed.

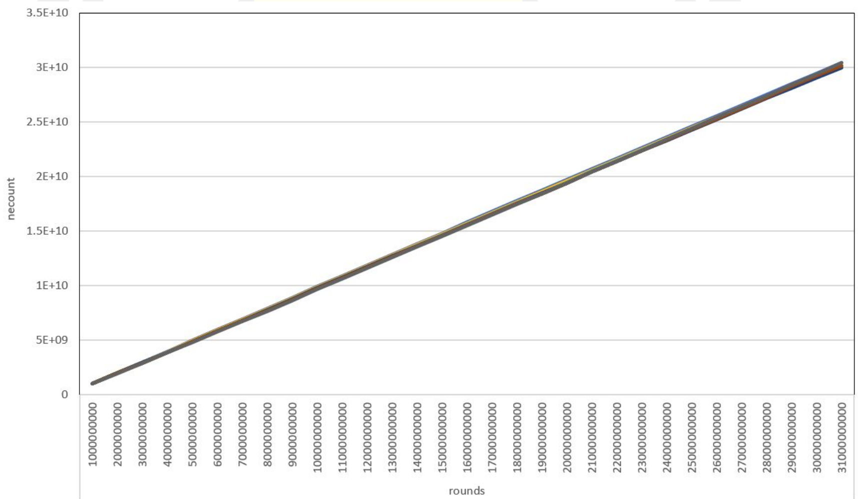


Figure. 6.3. NE Count.

Figure 6.3 shows that necount has a constant and incremental NE count. Starting from round 1 billion, the amount of NE Range 0 - 3 at 988,639,264 until rising continuously in the range 24 - 27 at 30,389,551,438

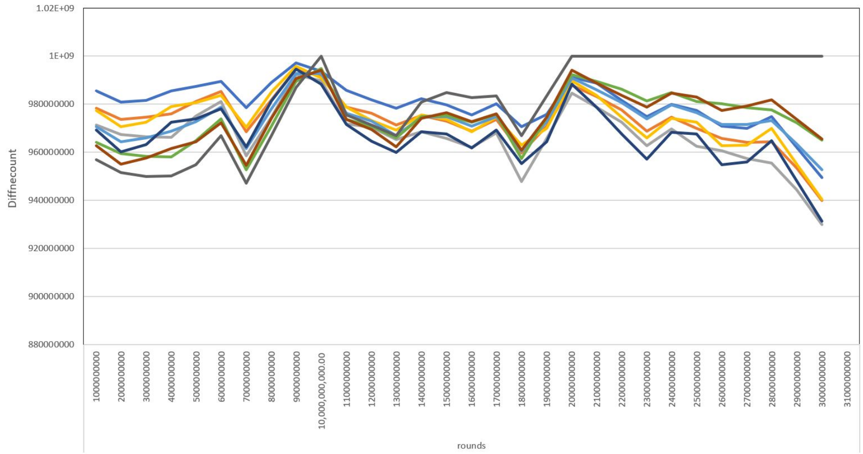


Figure. 6.4. Diff NE Count.

Figure 6.4 shows that when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

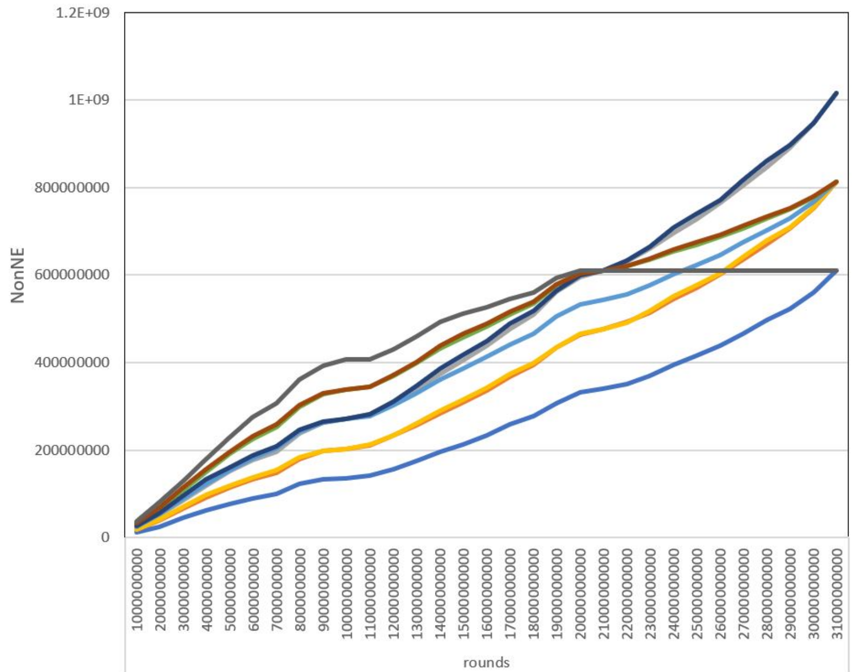


Figure. 6.5. NonNE.

Figure 6.5 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

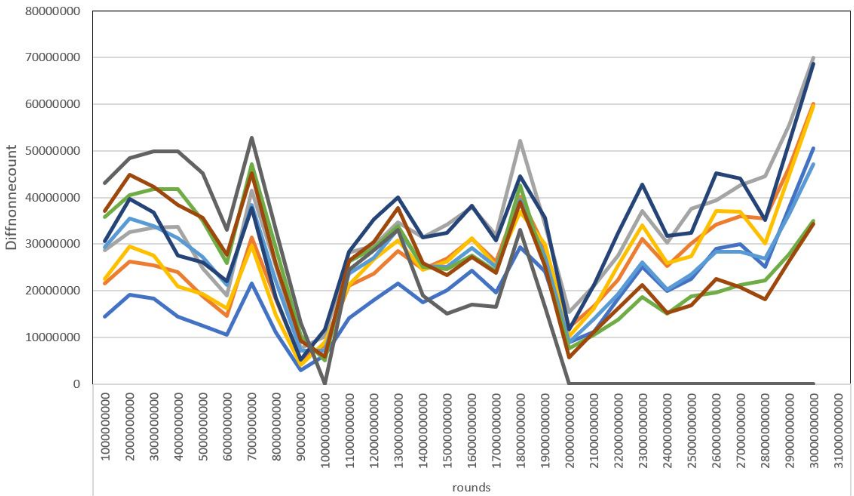


Figure. 6.6. Diff NonNE Count.

Figure 6.6 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

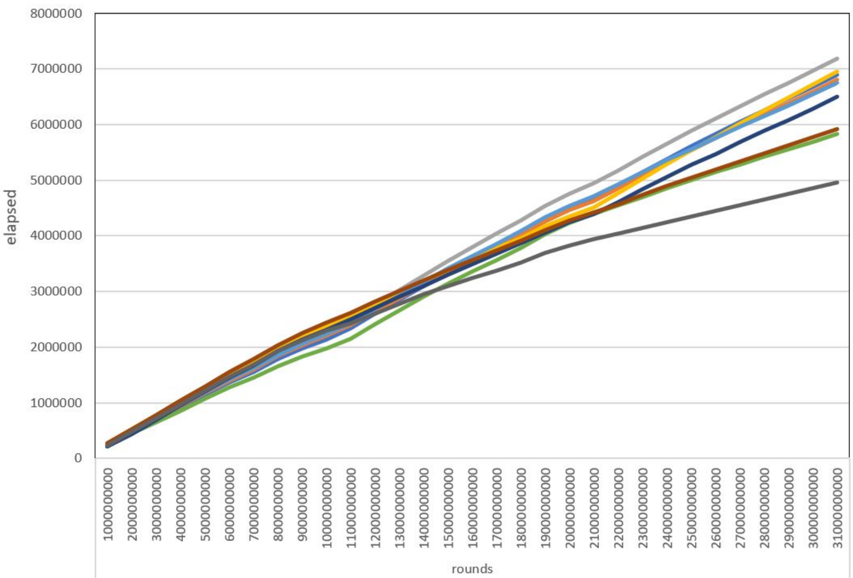


Figure. 6.7. Elapsed.

6.7.2 3p2s loop1

The result of the process of Overview 3p2s loop1. Divided into 5 parts as follows,

1. Results from finding NE Count Loop1.
2. Results from finding the difference NE Count Loop1.
3. Results from NonNE Count results Loop1.
4. Results of finding differences in NonNE Count Loop1.
5. Results from finding differences Elapsed Loop1.

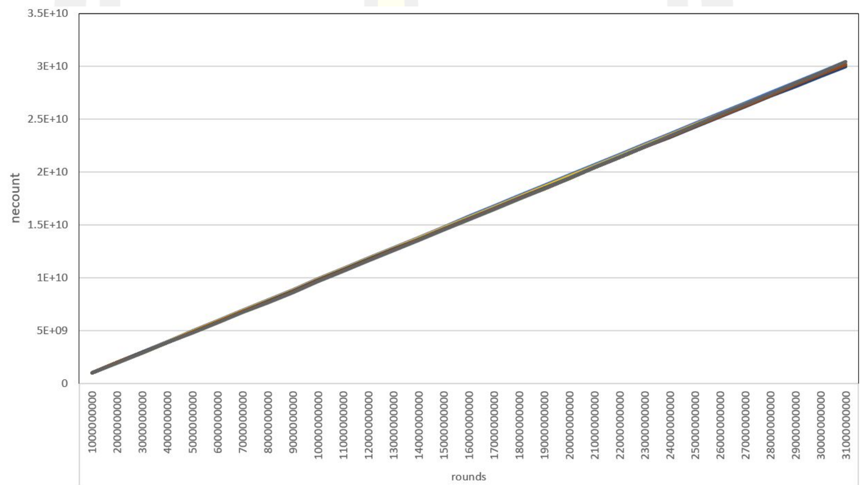


Figure. 6.8. NE Count Loop1.

Figure 6.8 shows that necount has a constant and incremental NE count. Starting from round 1 billion, the amount of NE Range 0 - 3 at 988,639,264 until rising continuously in the range 24 - 27 at 30,389,551,438

6.7.3 3p2s loop2

The result of the process of Overview 3p2s loop2. Divided into 5 parts as follows,

1. Results from finding NE Count Loop2.
2. Results from finding the difference NE Count Loop2.
3. Results from NonNE Count results Loop2.
4. Results of finding differences in NonNE Count Loop2.
5. Results from finding differences Elapsed Loop2.

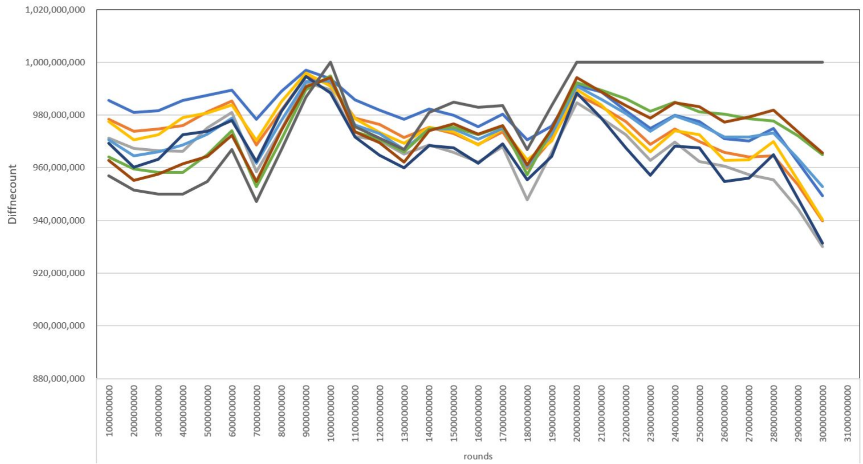


Figure. 6.9. Diff NE Count Loop1.

Figure 6.9 shows that when finding the difference in the number of NE in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

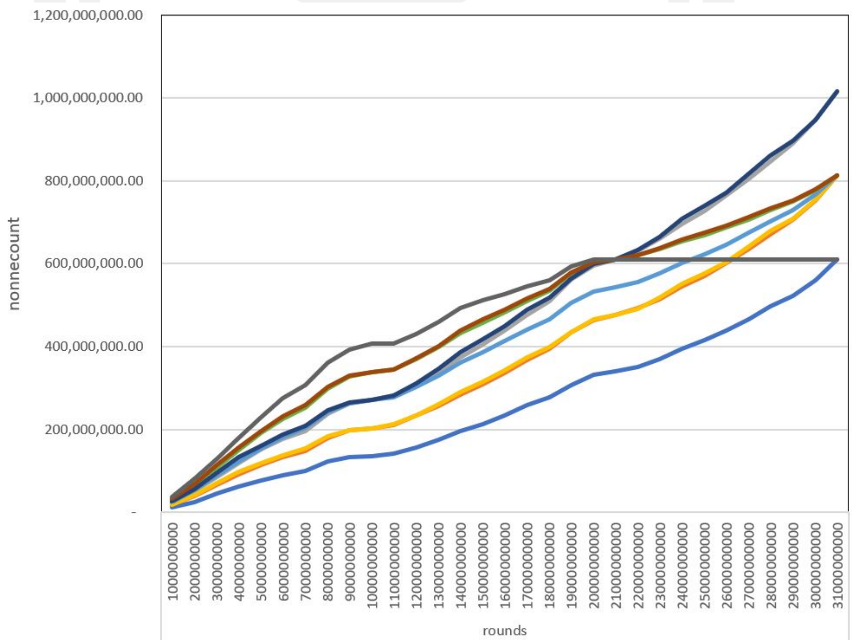


Figure. 6.10. NonNE Loop1.

Figure 6.10 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

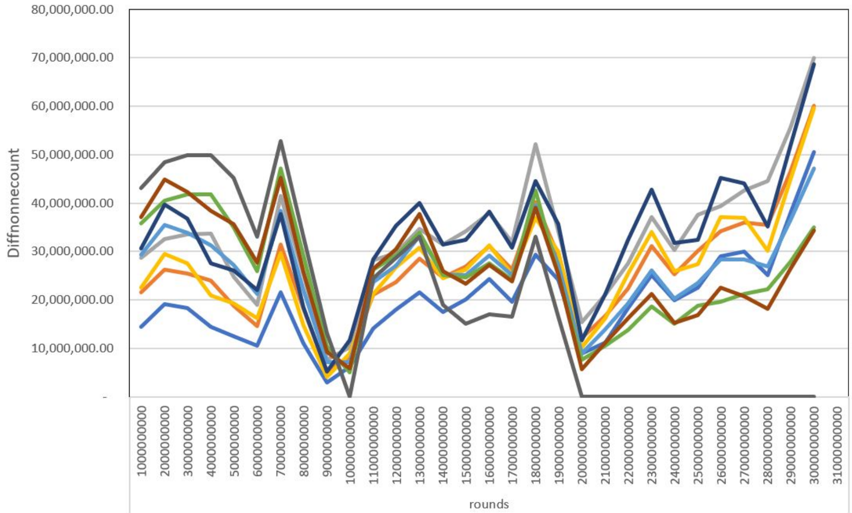


Figure. 6.11. Diff NonNE Count Loop1.

Figure 6.11 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

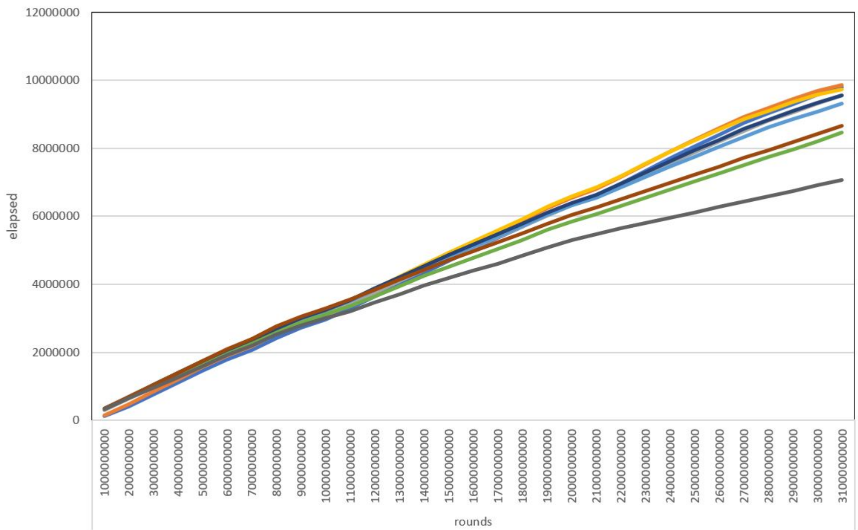


Figure. 6.12. Elapsed Loop1.

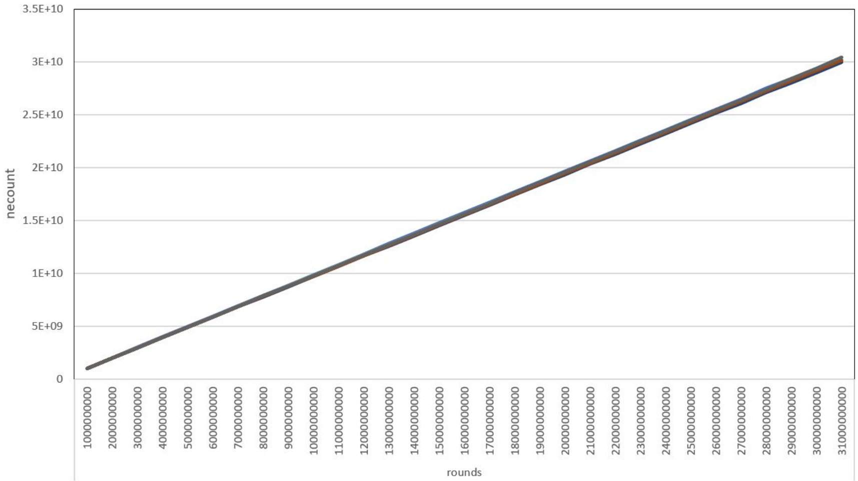


Figure. 6.13. NE Count Loop2.

Figure 6.13 shows that, when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

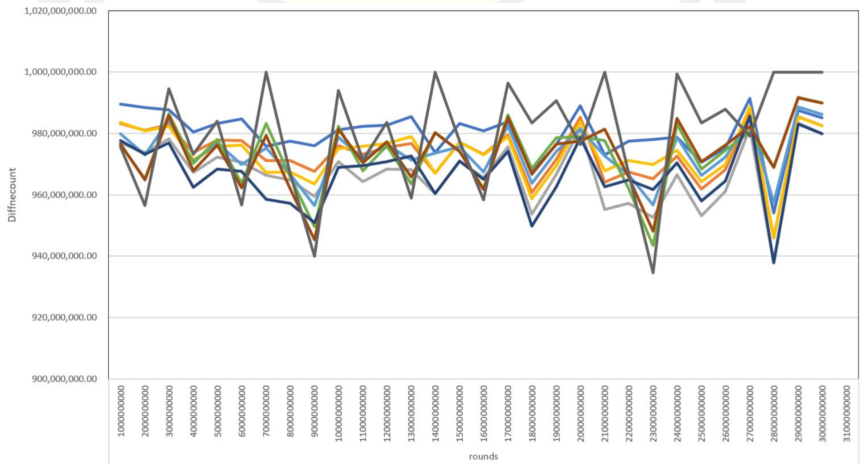


Figure. 6.14. Diff NE Count Loop2.

Figure 6.14 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

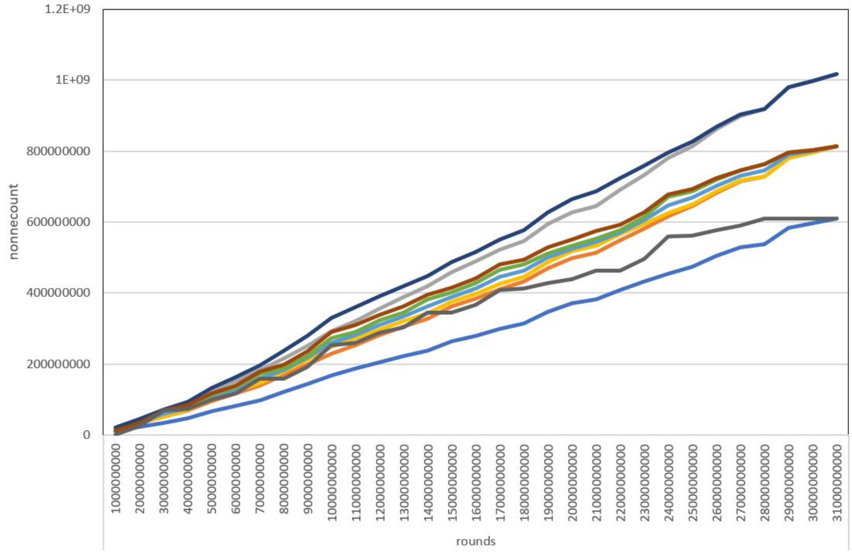


Figure. 6.15. NonNE Loop2.

Figure 6.15 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

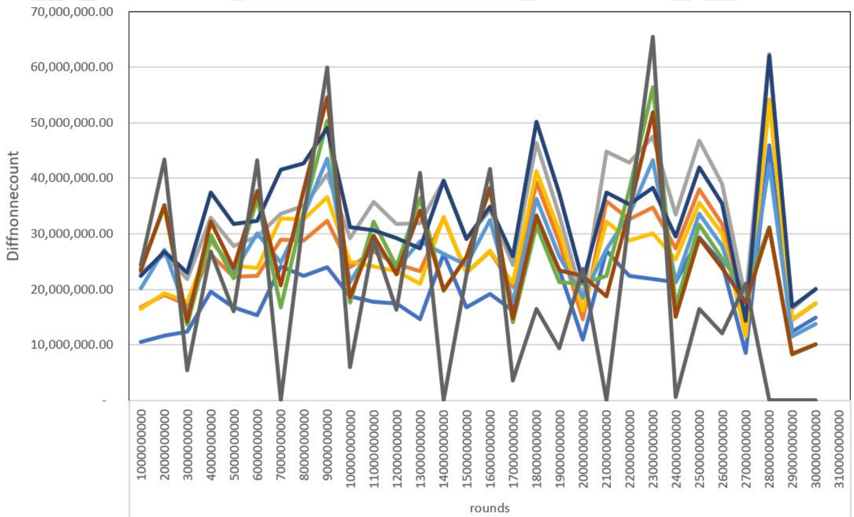


Figure. 6.16. Diff NonNE Count Loop2.

Figure 6.16 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

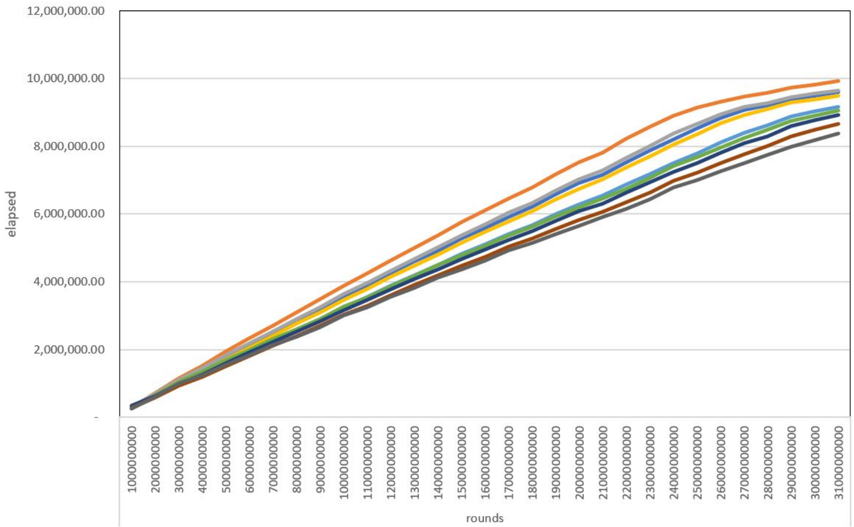


Figure. 6.17. Elapsed Loop2.

6.7.4 3p2s loop3

The result of the process of Overview 3p2s loop3. Divided into 5 parts as follows,

1. Results from finding NE Count Loop3.
2. Results from finding the difference NE Count Loop3.
3. Results from NonNE Count results Loop3.
4. Results of finding differences in NonNE Count Loop3.
5. Results from finding differences Elapsed Loop3.

6.7.5 3p2s loop4

The result of the process of Overview 3p2s loop4. Divided into 5 parts as follows,

1. Results from finding NE Count Loop4.
2. Results from finding the difference NE Count Loop4.
3. Results from NonNE Count results Loop4.
4. Results of finding differences in NonNE Count Loop4.
5. Results from finding differences Elapsed Loop4.

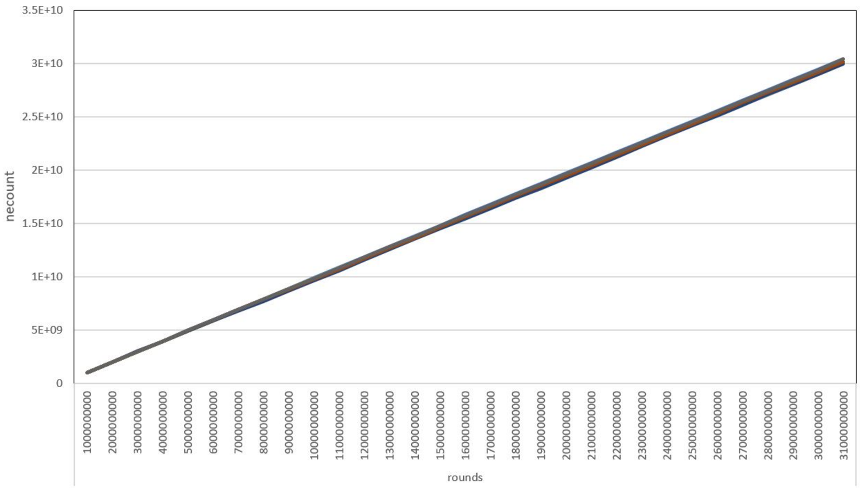


Figure. 6.18. NE Count Loop3.

Figure 6.18 shows that when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

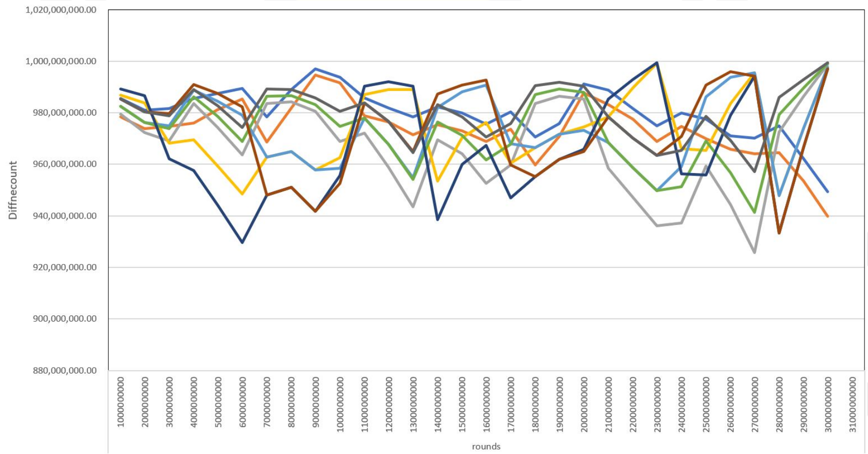


Figure. 6.19. Diff NE Count Loop3.

Figure 6.19 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

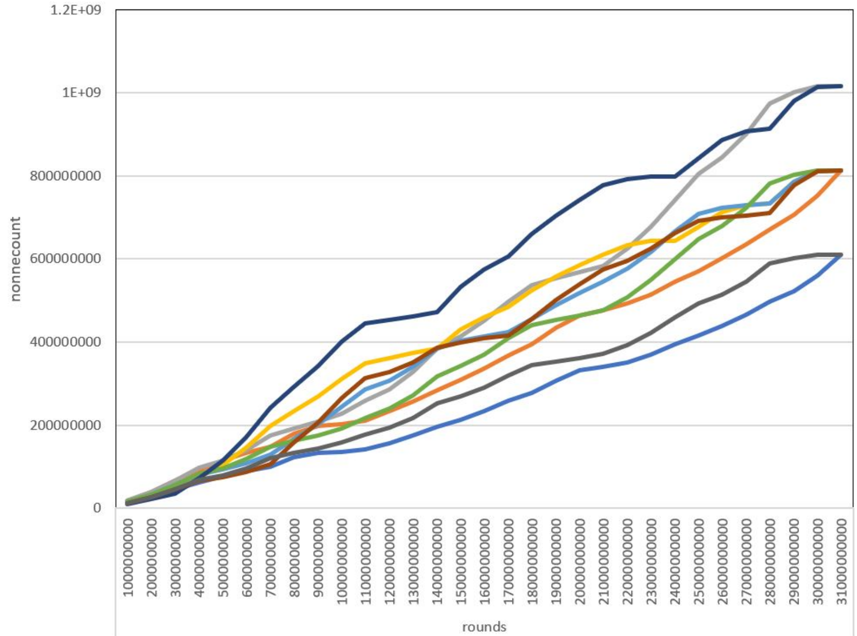


Figure. 6.20. NonNE Loop3.

Figure 6.20 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

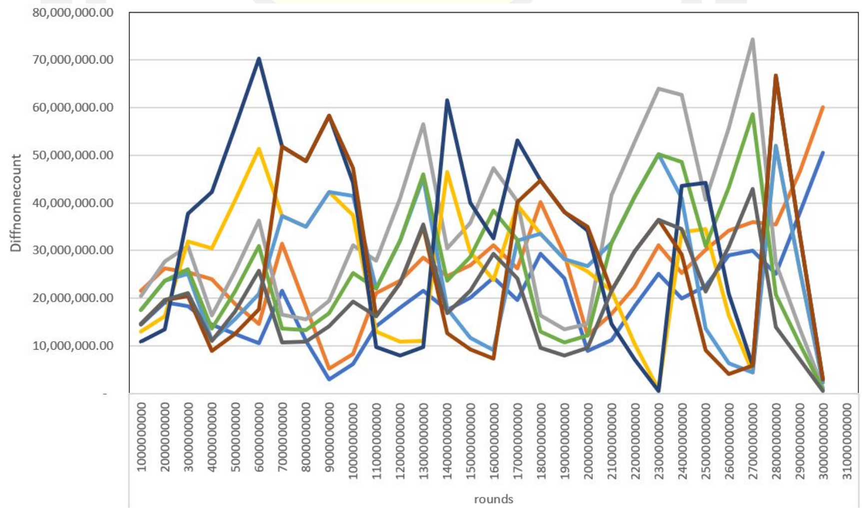


Figure. 6.21. Diff NonNE Count Loop3.

Figure 6.21 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

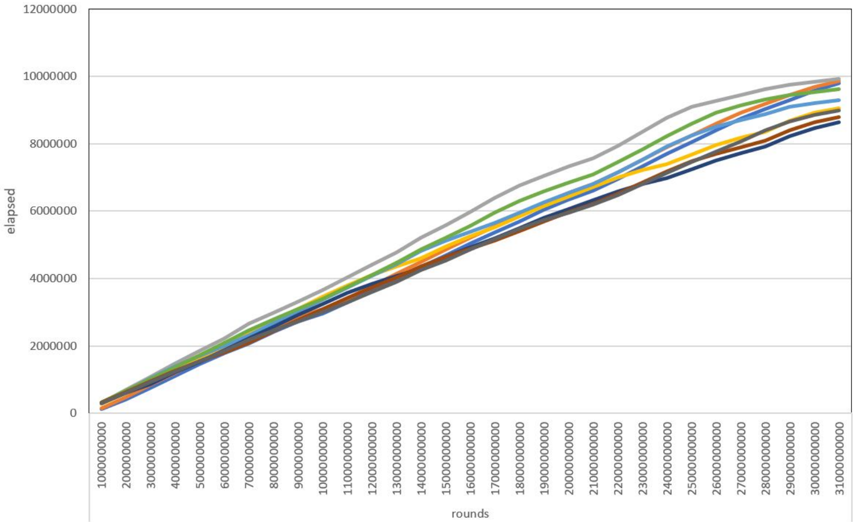


Figure. 6.22. Elapsed Loop3.

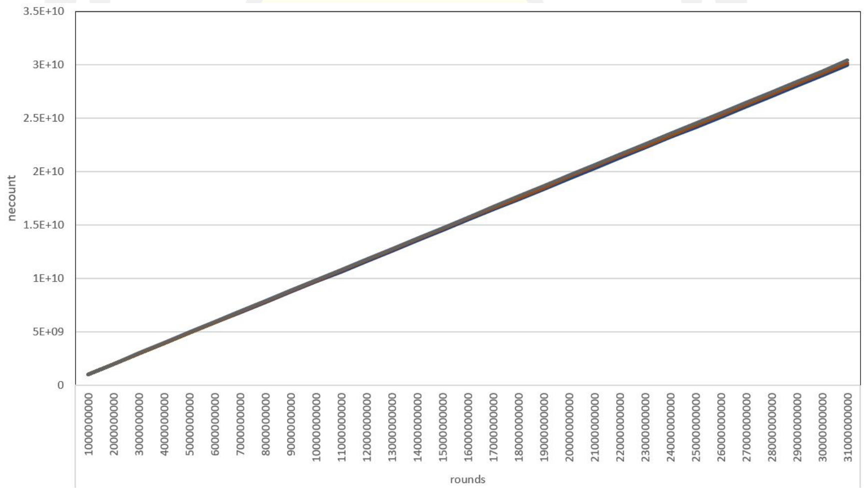


Figure. 6.23. NE Count Loop4.

Figure 6.23 shows then, when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

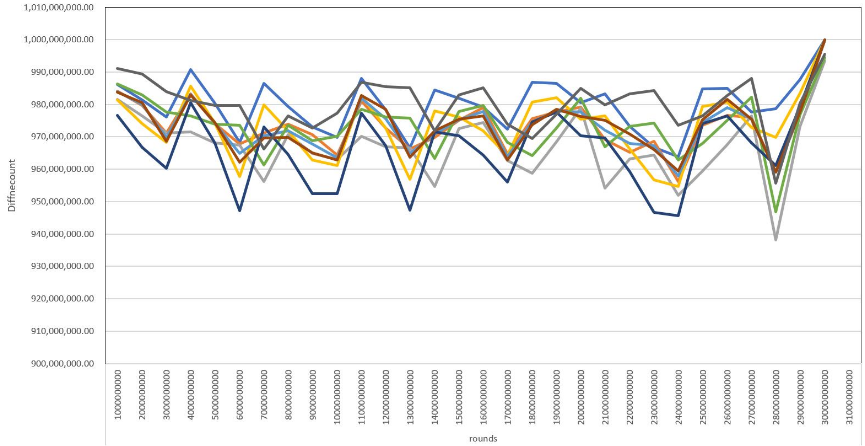


Figure. 6.24. Diff NE Count Loop4.

Figure 6.24 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

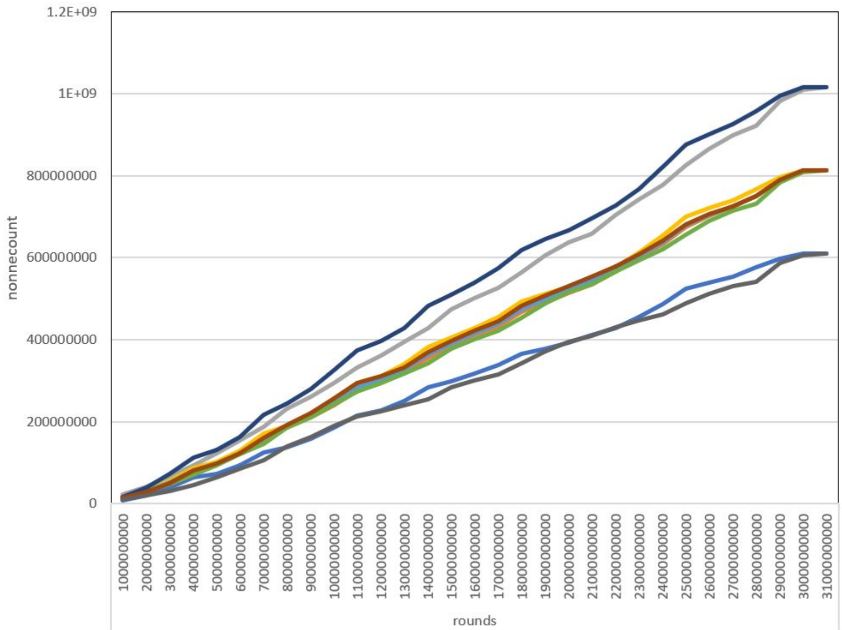


Figure. 6.25. NonNE Loop4.

Figure 6.25 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

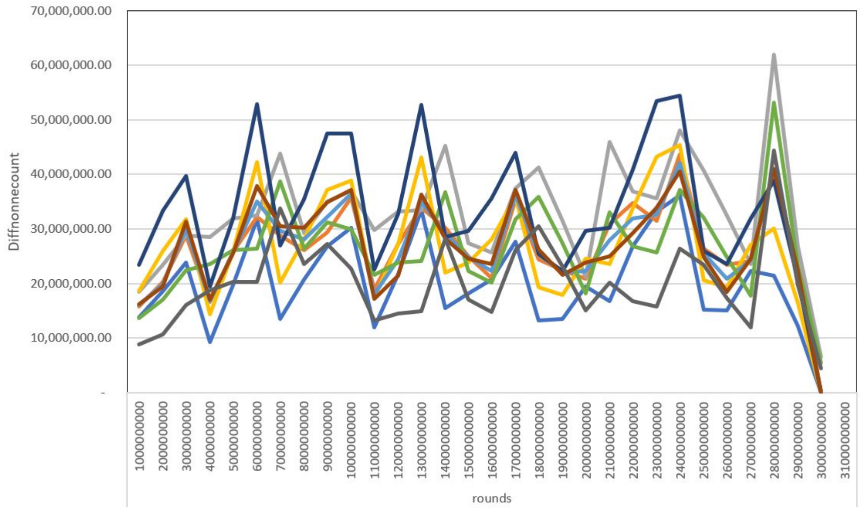


Figure. 6.26. Diff NonNE Count Loop4.

Figure 6.26 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

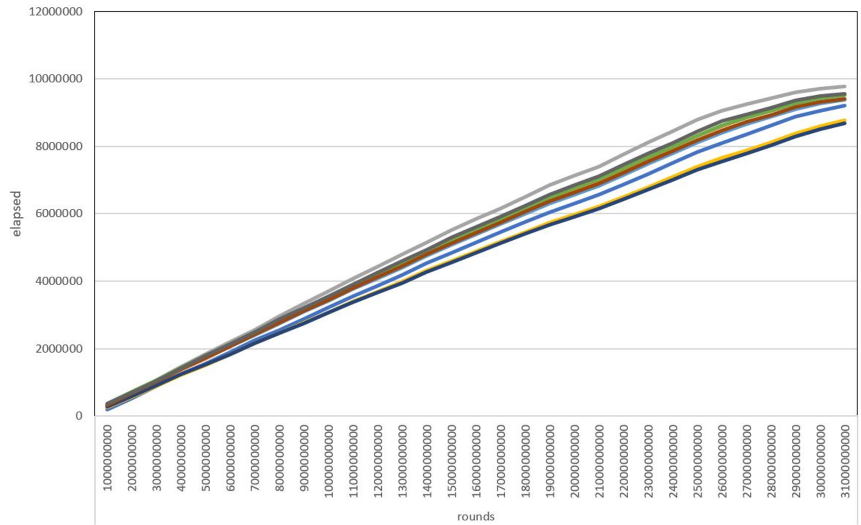


Figure. 6.27. Elapsed Loop4.

6.7.6 3p2s loop5

The result of the process of Overview 3p2s loop5. Divided into 5 parts as follows,

1. Results from finding NE Count Loop5.
2. Results from finding the difference NE Count Loop5.
3. Results from NonNE Count results Loop5.
4. Results of finding differences in NonNE Count Loop5.
5. Results from finding differences Elapsed Loop5.

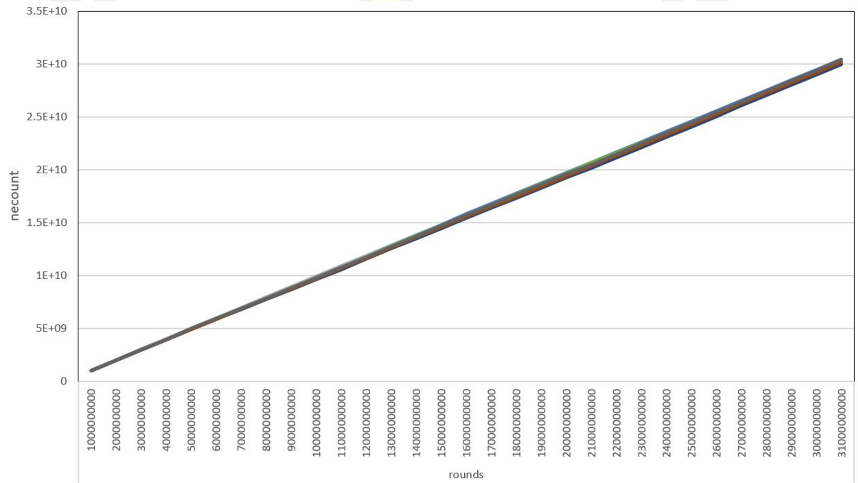


Figure. 6.28. NE Count Loop5.

Figure 6.28 shows that when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

6.7.7 3p2s loop6

The result of the process of Overview 3p2s loop6. Divided into 5 parts as follows,

1. Results from finding NE Count Loop6.
2. Results from finding the difference NE Count Loop6.
3. Results from NonNE Count results Loop6.
4. Results of finding differences in NonNE Count Loop6.
5. Results from finding differences Elapsed Loop6.

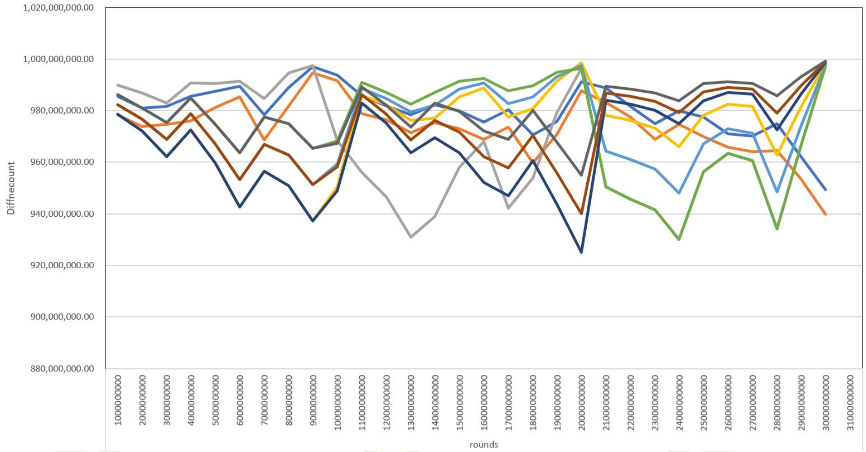


Figure. 6.29. Diff NE Count Loop5.

Figure 6.29 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

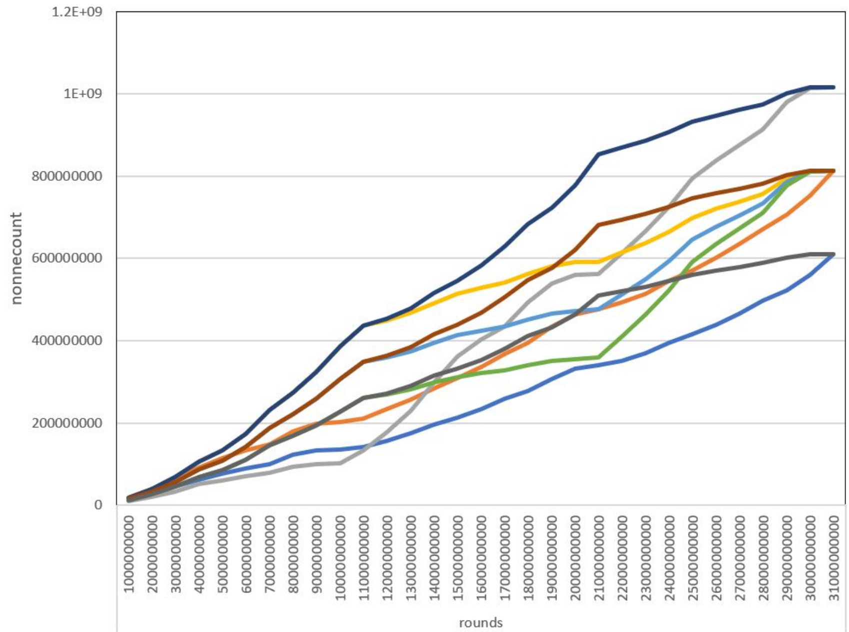


Figure. 6.30. NonNE Loop5.

Figure 6.30 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

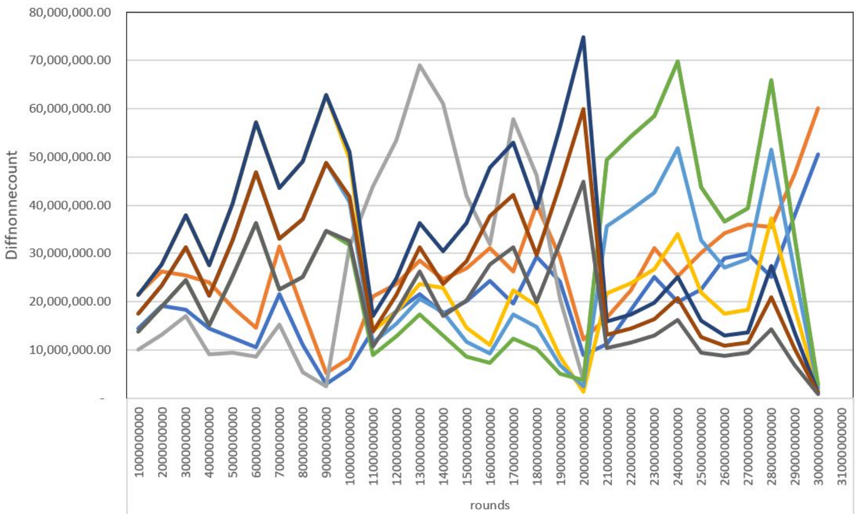


Figure. 6.31. Diff NonNE Count Loop5.

Figure 6.31 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

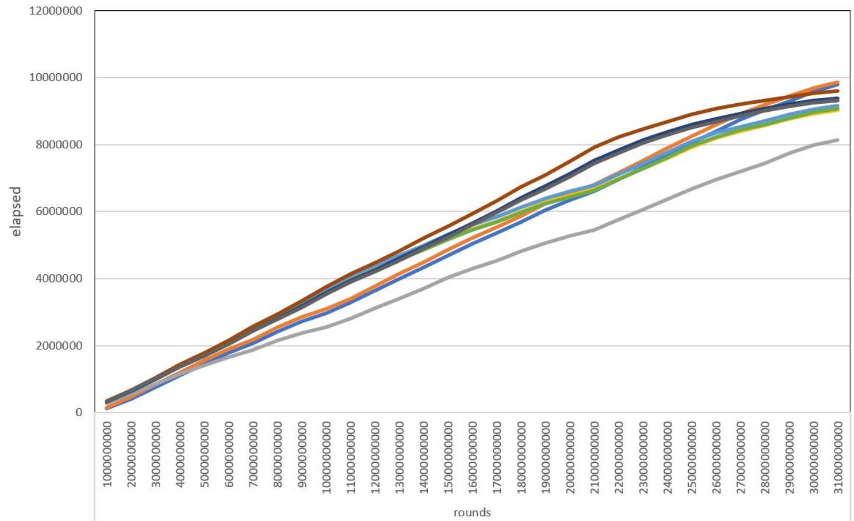


Figure. 6.32. Elapsed Loop5.

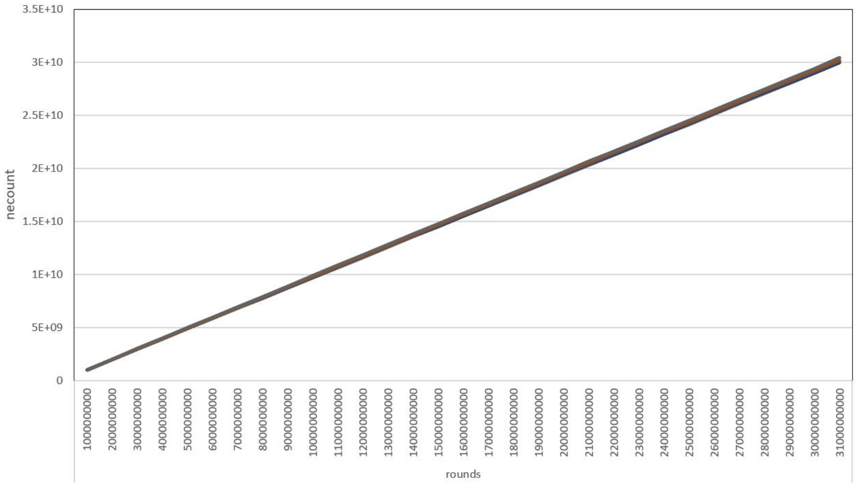


Figure. 6.33. NE Count Loop6.

Figure 6.33 shows that when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

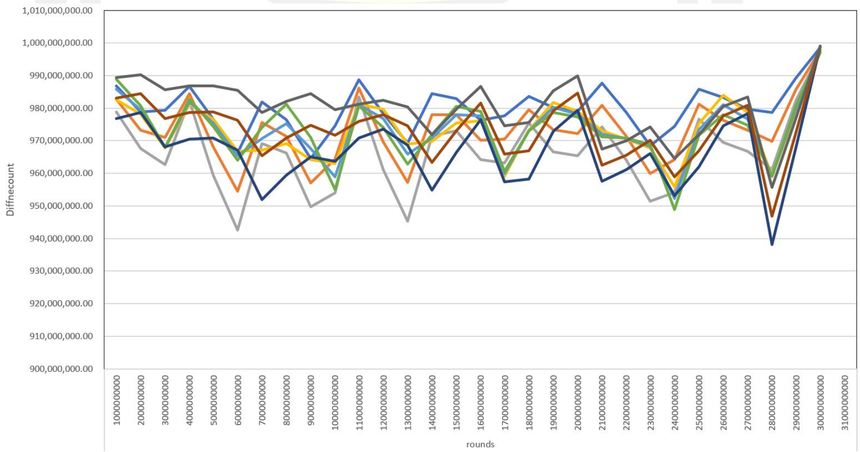


Figure. 6.34. Diff NE Count Loop6.

Figure 6.34 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

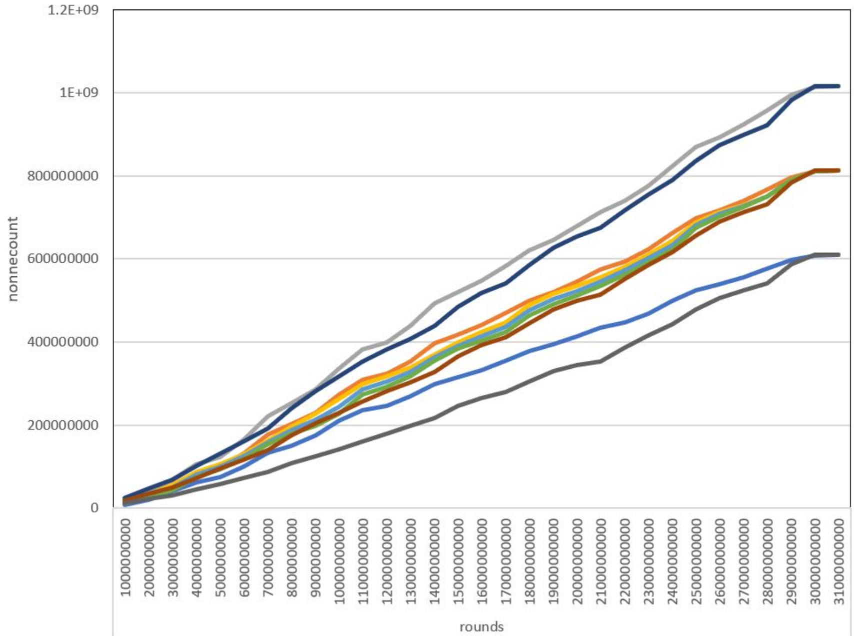


Figure. 6.35. NonNE Loop6.

Figure 6.35 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

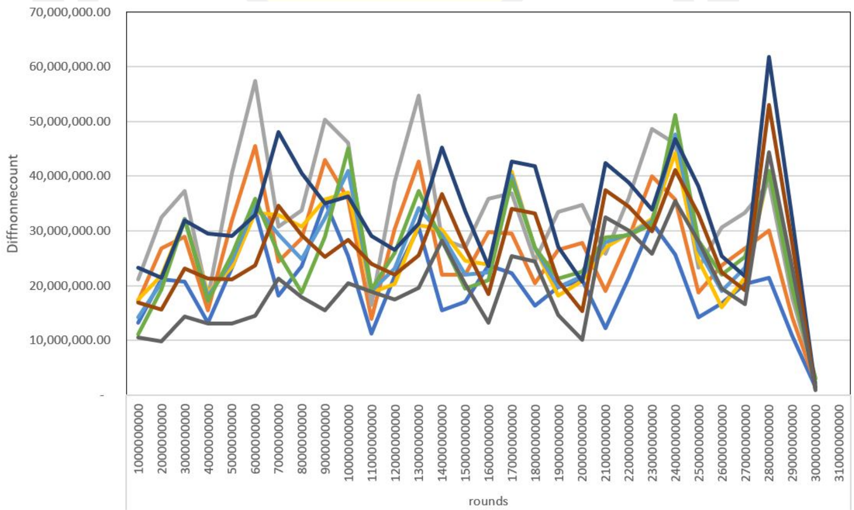


Figure. 6.36. Diff NonNE Count Loop6.

Figure 6.36 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

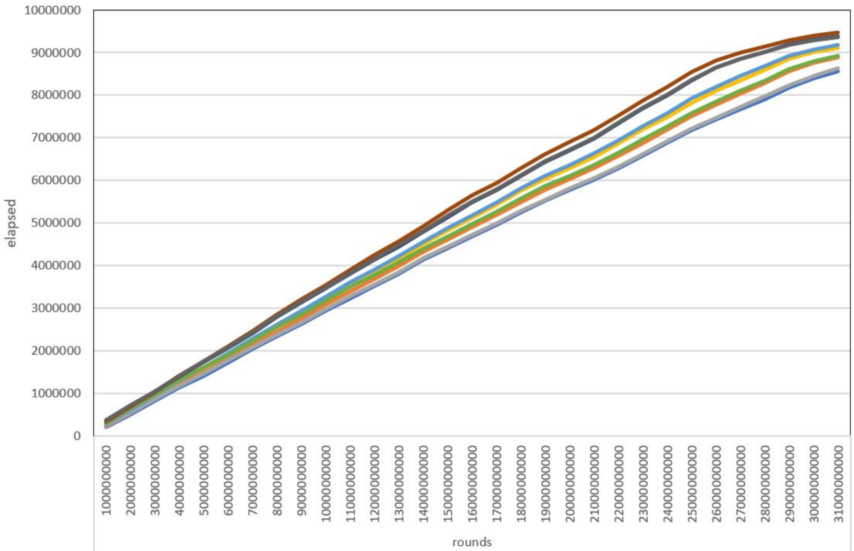


Figure. 6.37. Elapsed Loop6.

6.7.8 3p2s loop7

The result of the process of Overview 3p2s loop7. Divided into 5 parts as follows,

1. Results from finding NE Count Loop7.
2. Results from finding the difference NE Count Loop7.
3. Results from NonNE Count results Loop7.
4. Results of finding differences in NonNE Count Loop7.
5. Results from finding differences Elapsed Loop7.

6.7.9 3p2s loop8

The result of the process of Overview 3p2s loop8. Divided into 5 parts as follows,

1. Results from finding NE Count Loop8.
2. Results from finding the difference NE Count Loop8.
3. Results from NonNE Count results Loop8.
4. Results of finding differences in NonNE Count Loop8.
5. Results from finding differences Elapsed Loop8.

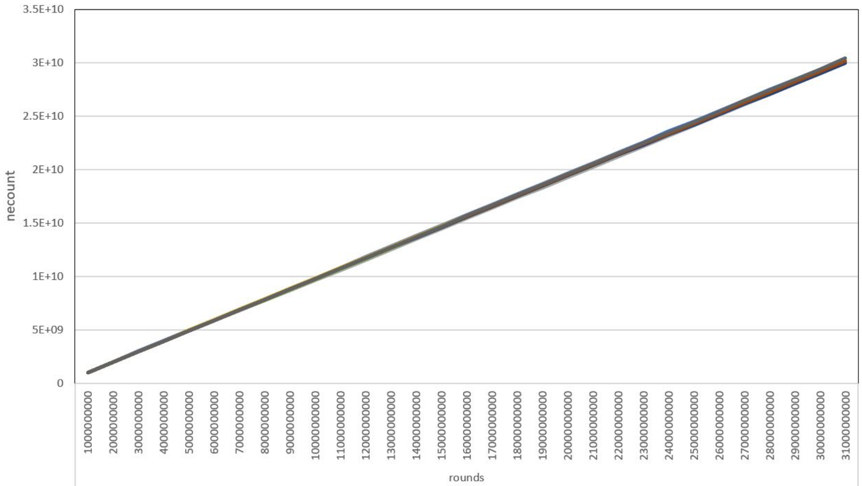


Figure. 6.38. NE Count Loop7.

From the Figure 6.38 shows then, when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

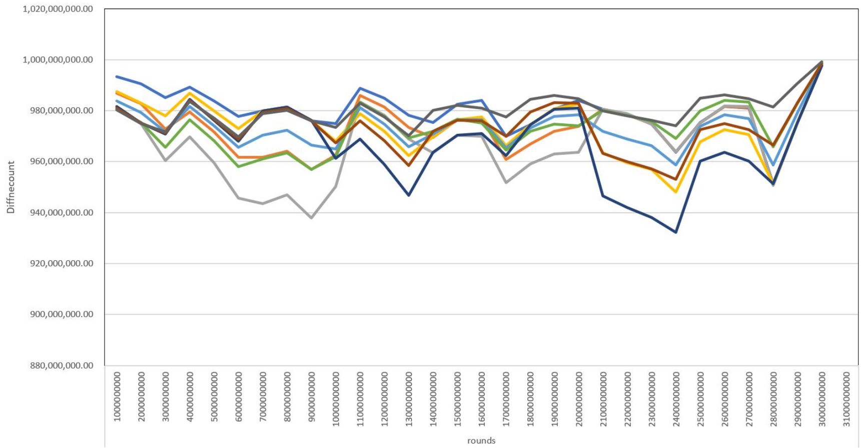


Figure. 6.39. Diff NE Count Loop7.

Figure 6.39 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

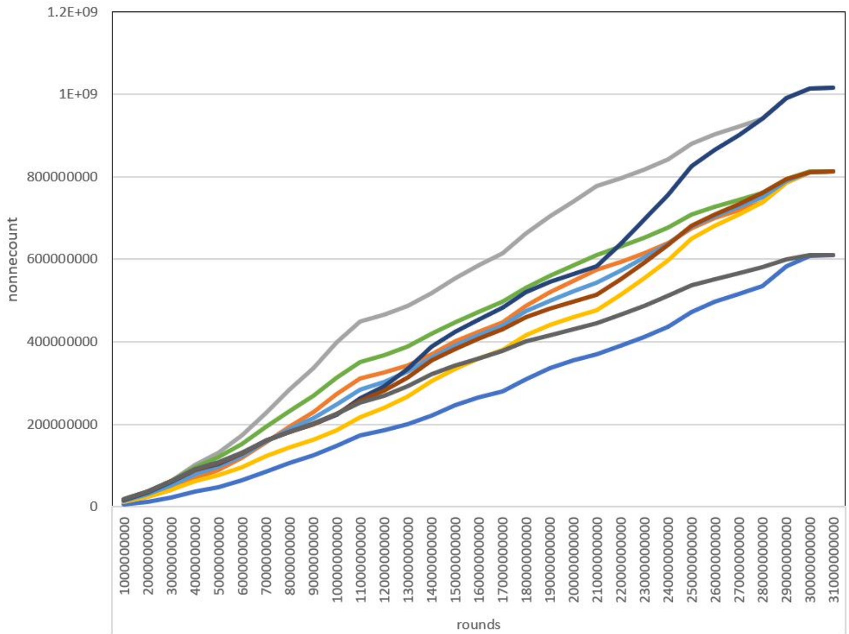


Figure. 6.40. NonNE Loop7.

Figure 6.40 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

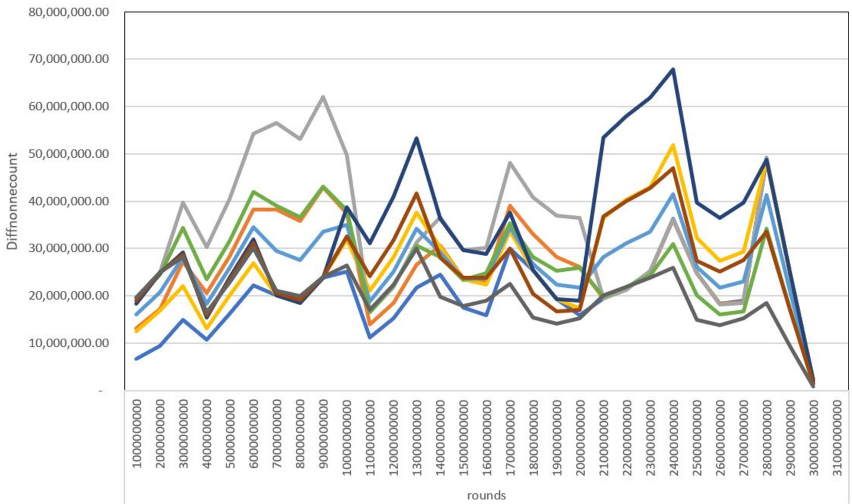


Figure. 6.41. Diff NonNE Count Loop7.

Figure 6.41 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

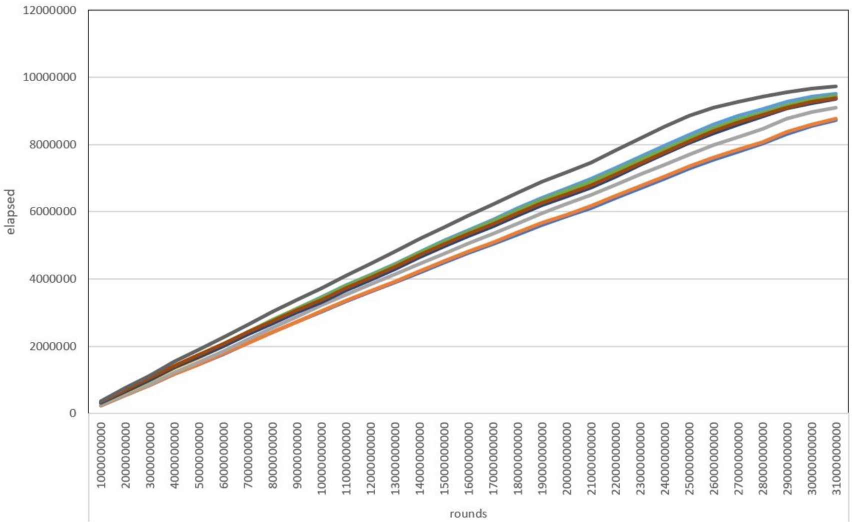


Figure. 6.42. Elapsed Loop7.

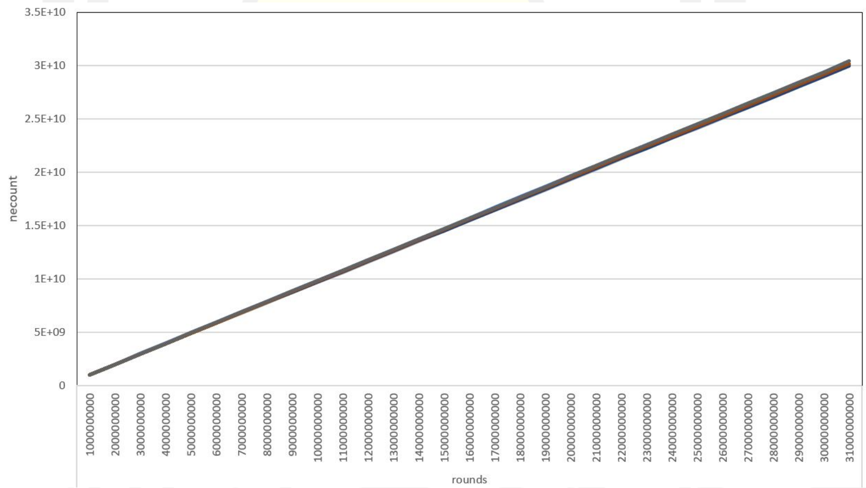


Figure. 6.43. NE Count Loop8.

Figure 6.43 shows that when finding the difference in the number of NEs in each rounds. The result is shown in the Diffnecount graph. The number of NE in the range 24 - 27 is Difference during round 10,000,000,000. The NE difference is 10,000,000,000. Then decreasing and increasing until 10,000,000,000 again at round 20,000,000,000 at a constant at 10,000,000,000 up to round 30,000,000,000.

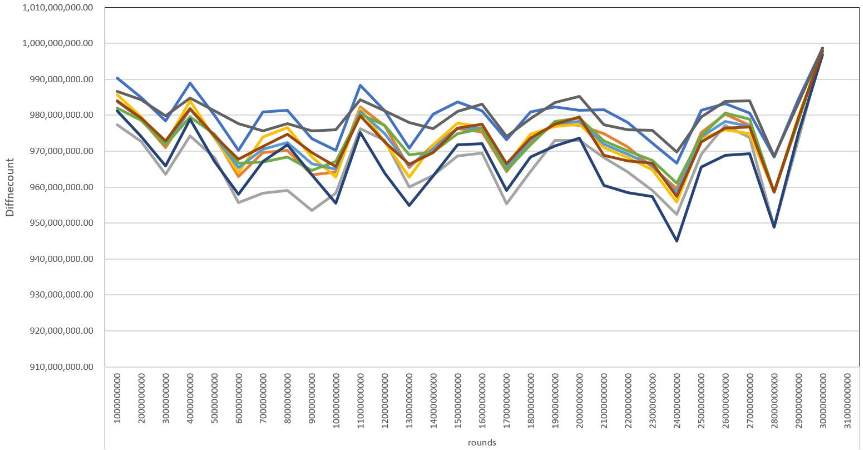


Figure. 6.44. Diff NE Count Loop8.
Figure 6.44 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

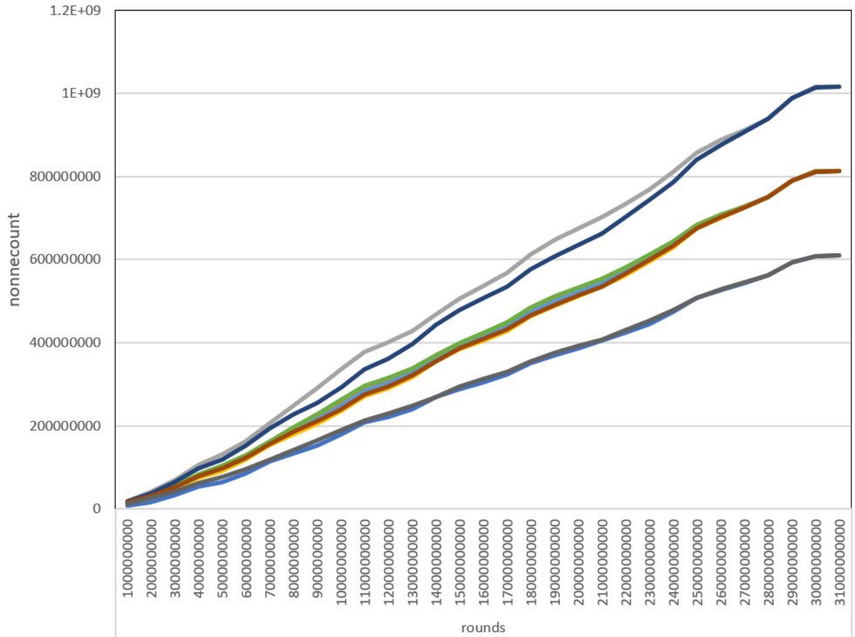


Figure. 6.45. NonNE Loop8.
Figure 6.45 shows that the number of Diff NonNE Count that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 43,026,045 up to round 20,000,000,000 At a constant at 0 to round 30,000,000,000.

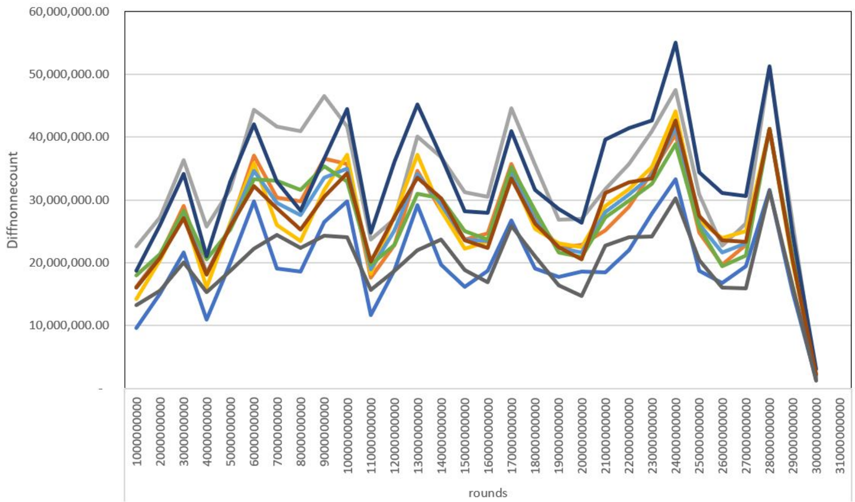


Figure. 6.46. Diff NonNE Count Loop8.

Figure 6.46 shows that the number of NonNE that occurred increasing steadily. But with a range of games during the period 24 - 27. Has a higher NnoNE count start point than every period starting from 38,335,680 up to round 20,000,000,000 At a constant at 610,448,562 to round 31,000,000,000.

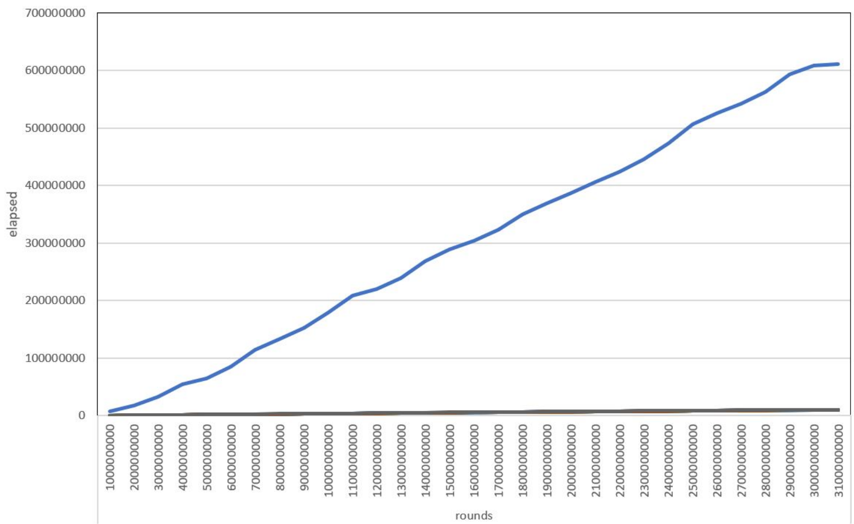


Figure. 6.47. Elapsed Loop8.

6.8 Conclusion

In this chapter we extensively explore a vast search space of generic strategic form game. Altogether, there are $27 \times 27 \times 27 \times 27 \times 27 \times 27 \times 27 \times 27 = 282,429,536,481$ games. We investigate these game and found there exists external relation of these games. In addition, we further explore the internal relationship by looking for Nash equillibrium in each of them. We categorize these games based on their external relation. Altogether, there are 35 cases. We found that there are 9 cases that there are certain patterns, or 25%. This shows that having extensively explore over 28 millions games, there exist certain patterns.

CHAPTER 7

CONCLUSION

7.1 Conclusion Chapter 2

It has been shown that the principle of computing Shapley value has been widely adopted and extended to real world applications for long time. In this research, we further investigate the results of applying the principle of Shapley value in wider domains. both in theoretical basis and real world application basis.

7.2 Conclusion Chapter 3

In this chapter, we explore the final payoffs of agents whether they are affected by any pattern of coalition values under Shapley value. We have five coalition value distribution patterns, namely STA, INC, DEC, CAP and CUP. As we can see, the final agents payoffs are still in the same trends as their original values. However, the average payoffs of agents in coalition of different sizes are affected by these patterns.

7.3 Conclusion Chapter 4

Based on [40, 59] where only certain games are considered, we found that with our setting where resources are given of agents in various trends, payoffs of agents differ from trends of resources. In other words, the main factor that controls the trend of agents payoffs are both technology matrix and trend of resources.

7.4 Conclusion Chapter 5

We study non-cooperative bakery game. A wide range of amount of resources into 5 trends. Trend 1 is used as a reference. Trend 2, 3 and Trend 4, 5 are diagonally similar. Given certain technology matrix and price functions, we find that within our settings agents' strategies remain unchanged even though resources vary upto 75%. Furthermore, agents' payoffs changes relatively small. In the future, this research can be extended to consider more complex situations with more details. While there are a small number of agents and actions are used in this research, there should be more agents and actions involved. Furthermore, there could be algorithms working on other aspects, including efficiency, etc.

7.5 Conclusion Chapter 6

In this chapter we extensively explore a vast search space of generic strategic form game. Altogether, there are $27 \times 27 \times 27 \times 27 \times 27 \times 27 \times 27 \times 27 = 282,429,536,481$ games. We investigate these game and found there exists external relation of these games. In addition, we further explore the internal relationship by looking for Nash equillibrium in each of them. We categorize these games based on their external relation. Altogether, there are 35 cases. We found that there are 9 cases that there are certain patterns, or 25%. This shows that having extensively explore over 28 millions games, there exist certain patterns.

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พหุ่น ปญ ศิโต ชีเว