A study on bi-interior hyperideals in hypersemigroups

Jirayus Tangtragoon

A Thesis submitted in partial fulfillment of the requirements for
the degree of Master of Science in Mathematics
at Mahasarakham University
October 2024

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The examining committee has unanimously approved this thesis, submitted by Mr. Jirayus Tangtragoon, as a partial fulfillment of the requirements for the Master of Science in Mathematics at Mahasarakham University.

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Jirayus Tangtragoon

พหูน ปณ ฑิโต ชีเว

ชื่อเรื่อง การศึกษาไบอินทิเรียไฮเปอร์ไอดีลในกึ่งกรุปไฮเปอร์
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บทคัดย่อ

ในการวิจัยนี้ ผู้วิจัยได้นำเสนอแนวคิดของไบอินทิเรียไฮเปอร์ไอดีลบนไฮเปอร์กึ่งกรุปและศึกษาสมบัติพื้นฐานบางประการ เช่น การอินเตอร์เซกชัน สับไบอินทิเรียไฮเปอร์ไอดีล และผลคูณตรง รวมถึงนำเสนอความสัมพันธ์ระหว่างไบอินทิเรียไฮเปอร์ไอดีลกับไฮเปอร์ไอดีลชนิดอื่นๆ บนไฮเปอร์กึ่งกรุป

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ABSTRACT

In this thesis, we introduce the notion of bi-interior hyperideals in hypersemigroups and investigate some basic properties such as the intersection, subbi-interior hyperideal and direct product. Moreover, we present some relations between bi-interior hyperideals and other hyperideals.

Keywords : Hypersemigroup, simple hypersemigroup, regular hypersemigroup, hyperideal, quasi-hyperideal, bi-hyperideal, interior hyperideal, bi-interior hyperideal.

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CHAPTER 1

INTRODUCTION

1.1 Background

The hyperstructure theory was first introduced by Marty in 1934 [13]. Hyperstructures have been used to study several branches of science such as mathematics, computer science, biology, chemistry, etc. (see [15, 16, 17, 18, 19]). Marty extended the composition of two elements in algebraic structure to be a set. The concept of hyperstructure can be seen as a generalization of algebraic structure such as rings, groups and semigroups. Therefore many investigation of algebraic structure can be shifted to hyperstructures.

In this thesis, we focus on the extended version of semigroups, the concept of hypersemigroups (semihypergroups, multisemigroups). The concepts of hypersemigroups was developed by many aspects of hypergroups. Different perspectives on hypersemigroups were estrablished (see [2, 5, 7, 8]). Many problems in semigroups can be considered in terms of hypersemigroups. Jafarabadi et al. [7, 27] introduced the concept of simple hypersemigroups. They considered the product and quotient of simple hypersemigroups if the resulting hypersemigroups are simple.

Hasankhani [5] introduced the notions of left (resp. right) hyperideals and examined their preliminary properties. They studied Green's relations of hypersemigroups. It turns out that the notions of various kinds of hyperideals in hypersemigroup have been interesting to many researchers since hyperideals are essential in investigating Green's relations of hypersemigroups. Changphas and Davvaz [8] introduced the notions of bi-hyperideals and quasi-hyperideals in ordered hypersemigroups.

The concept of bi-interior ideal in semigroup was introduced by M.M.K. Rao [1]. He illustrated some basic properties. Moreover, he investiged the relation between bi-interior ideals and other types of ideals, namely left (resp., right) hyperideals, two side hyperideals, bi-hyperideals, interior hyperideals and quasi-hyperideals in semigroups, in simple semigroups and in regular semigroups.

The other hyperideals in hypersemigroups were defined in a more general setting of hypersemigroups. For example, bi-hyperideals and quasi-hyperideals in ordered hypersemigroups were introduced by Changphas and Davvaz in [8]. They used bi-hyperideals to characterized intra-regular ordered hypersemigroups. The notion of (fuzzy) interior hyperideals in ordered semigroups were introduced independently in 2016. Tang et al. [11] considered the normality of fuzzy interior hyperideals and gave several related characterize simple ordered hypersemigroups.

The conceptions of bi-hyperideals and interior hyperideals defined in ordered hypersemigroups with the equality relation are the notions in hypersemigroups since every ordered hypersemigroups with the equality relation can be regarded as a hypersemigroup. This means that hypersemigroups notions of left (resp., right, quasi-, bi-, interior) hyperideals play improtant roles in exploring hypersemigroups. Recently, Lekkoksung et al. [12] defined more generalized notions of bi-hyperideals and interior hyperideals in hypersemigroups. They introduced the concept of (m,n)-hyperideals and n-interior hyperideals in hypersemigroups through the notion of ideal element. Furthermore, these hyperideals were used to characterize many classes of hypersemigroups.

We can see the significance of hyperideals in hypersemigroups from the mentioned above. These implications motivate us to define a new hyperideal, which is a generalization of the earlier notions. In chapter 2 we recalled some fundamental hypersemigroup knowledge and some hyperideals. Moreover, we introduce the concept of bi-interior hyperideals in hypersemigroups. In chapter 3, the general properties of bi-interior hyperideals are given such as intersection, subbi-interior hyperideal and direct product. We illustrate that bi-interior hyperideals are generalizations of bi-hyperideals and interior hyperideals. We give an example to show how we differentiated this concept from the earlier concepts. In chapter 4, the relationships between bi-interior hyperideals and others hyperideals are provided.

In chapter 5, researcher was interested to expand the idea of fundamental properties and the applications of bi-interior hyperideals in hypersemigroups, in simple hypersemigroups and in regular hypersemigroups.

1.2 Objective of the research

The purposes of the research are:

1. To defined the definition of bi-interior hyperideals and investigate some properties.
2. To defined the direct product of bi-interior hyperideals and investigate some properties.
3. To study the relationship between bi-interior hyperideals and other hyperideals.

1.3 Procedure of the research

The research procedure of this thesis consists of the following steps:

1. Review the literature of semigroups and ideals.
2. Review the literature of bi-interior ideals in semigroups.
3. Review the literature of fundamental properties in hypersemigroups.
4. Review the literature of hyperideals in hypersemigroups.
5. Perform research.
6. To compile and disseminate a research publication.

1.4 Scope of the study

The scopes of the study are: study some fundamental properties of bi-interior hyperideals in hypersemigroups and study about the subbi-interior hyperideals and direct product. Lastly, we study about the relation between bi-interior hyperideals and other hyperideals.

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1.5 Antipicated outcomes

The goals of this thesis are to archieve the following outcome.

1.5.1 To obtain the definition of bi-interior hyperideals in hypersemigroups.

1.5.2 To obtain some properties of bi-interior hyperideals, namely the intersection, subbi-interior hyperideals and direct product.

1.5.3 To obtain the relation between bi-interior hyperideals and other hyperideals in hypersemigroups, simple hypersemigroups and regular hypersemigroups.

1.6 Outline of the thesis

Some important preliminaries and basic results are given in Chapter 2. We start with the definition of semigroup and related properties in section 2.1. In section 2.2, we provide definition of hypersemigroups and related properties.

In Chapter 3, we give the definition of bi-interior hyperideals, examples and the intersection in section 3.1. In section 3.2, we give the notion of subbi-interior hyperideals. In section 3.3, we define the direct product of hypersemigroups and examine their properties.

In Chapter 4, we study the relations between bi-interior hyperideals and other hyperideals, namely left (resp., right) hyperideals, two-side hyperideals (hyperideals), quasi-hyperideals, bi-hyperideals and interior hyperideals in hypersemigroups, in simple hypersemigroups and in regular hypersemigroups.

Lastly, in Chapter 5, we summarize the results of this thesis.

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CHAPTER 2

PRELIMINARIES

In this chapter, we will give some definitions, notations and some useful results of semigroups and hypersemigroups.

2.1 Semigroups

The essential properties and associated characteristics of semigroups as follows.

Definition 2.1.1. [20] A *semigroup* S is a groupoid $(S;*)$ where a binary operation * on S is associative.

Definition 2.1.2. [20] An element z of a semigroup $(S;*)$ is called a $left\ zero\ element$ of S if $zx = z$ for all $x \in S$ and called z is a *right zero element* of S if $xz = z$ for all $x \in S$.

If z is both a left and a right zero element of S. We called z is a *zero element* of S.

Definition 2.1.3. [20] Let $(S;*)$ and $(T;\circ)$ be semigroups. The *diect product* of S and T denoted by $S \times T$, and is defined by

$$S \times T = \{(s,t) \mid s \in S\ and\ t \in T\}.$$

We defined a binary operation $\odot$ on $S \times T$ by;

$$(a,b) \odot (x,y) = (a * x, b \circ y),$$

for any $(a,b), (x,y) \in S \times T$.

Definition 2.1.4. [9] The *direct product* of the family of semigroups $\{S_i\}_{i\in I}$ is the Cartesian product $S = \prod_{i\in T} S_i$ with operation $\cdot_{S_i} : S_i \times S_i \to S_i$, for which the product of two functions $f, g \in S$ is the function $f \cdot g : I \to \bigcup_{i\in T} S_i$ given by

$$(f \cdot g)(i) = f(i) \cdot_{S_i} g(i).$$

Proposition 2.1.5. [9] The direct product of two semigroups S and T is a semigroup. The direct product of a family of semigroups $\{S_i\}_{i\in I}$ is a semigroup.

Definition 2.1.6. [20] Let $(S;*)$ be a semigroup and $\phi \neq T \subseteq S$, we called T is a $subsemigroup$ of $(S;*)$ if $(T;*)$ is a semigroup and denoted by $T \leq S$.

Definition 2.1.7. [20] Let T_i be a semigroup of S for all $i \in I$ where I is an index set. If $\bigcap\limits_{i \in I} T_i \neq \phi$, then $\bigcap\limits_{i \in I} T_i \leq S$.

Definition 2.1.8. [20] For any subset X of a semigroup S, we defined

$$\langle X \rangle := \bigcap \{A \mid X \subseteq A \ \ and \ \ A \leq S\}.$$

We called $\langle X \rangle$ is a $subsemigroup\ of\ S\ generated\ by\ X$.

Definition 2.1.9. [20] Let S be a semigroup and $\phi \neq A \subseteq S$. We called A is a $left\ ideal$ of S if $SA \subseteq A$ and called A is a $right\ ideal$ of S if $AS \subseteq A$.

If A is both a left and a right ideal of S. We called A is a $two-side\ ideal$ or $ideal$ of S, that is $SA \subseteq A$ and $AS \subseteq A$.

Proposition 2.1.10. [20] Let S be a semigroup. Then

(1) If A is a left ideal of S, then A is a subsemigroup of S.

(2) If A is a right ideal of S, then A is a subsemigroup of S.

(3) If A is an ideal of S, then A is a subsemigroup of S.

Theorem 2.1.11. [20] Let S be a semigroup, I be an index set and $\{A_i \mid i \in I\}$ be a family of left (resp., right) ideals of S, then we obtain

(1) $\bigcap\limits_{i \in I} A_i$ is a left (resp., right) ideal of S, if $\bigcap\limits_{i \in I} A_i \neq \phi$.

(2) $\bigcup\limits_{i \in I} A_i$ is a left (resp., right) ideal of S.

Theorem 2.1.12. [20] Let S be a semigroup, I be an index set and $\{A_i \mid i \in I\}$ be a family of ideals of S, then we obtain

(1) $\bigcap\limits_{i \in I} A_i$ is an ideal of S, if $\bigcap\limits_{i \in I} A_i \neq \phi$.

(2) $\bigcup\limits_{i \in I} A_i$ is an ideal of S.

Definition 2.1.13. [20] Let S be a semigroup without zero element, we called I is a $proper\ left$ (resp., $right, two-side$) ideal of S if $I \neq S$.

If S contains zero element, we called I is a proper left (resp., right, two-side) ideal of S if $I \neq \{0\}$ and $I \neq S$.

Definition 2.1.14. [20] Let S be a semigroup and $\phi \neq A \subseteq S$. We called A is a $bi-ideal$ of S, then the following statements hold;

(1) A is a subsemigroup of S.

(2) $ASA \subseteq A$.

Definition 2.1.15. [20] Let S be a semigroup without zero element, we called S is a $left\ simple$ (resp., $right\ simple$, $simple$) semigroup if S is the only left (resp., right, two-side) ideal of S.

Definition 2.1.16. [21] Let S be a semigroup . A non-empty subset Q of S is called a $quasi-ideal$ of S if $SQ \cap QS \subseteq Q$.

Definition 2.1.17. [22] A subsemigroup A of a semigroup S is called an $interior\ ideal$ of S if $SAS \subseteq A$.

Definition 2.1.18. [1] A non-empty subset B of a semigroup S is said to be $bi-interior\ ideal$ of S if $SBS \cap BSB \subseteq B$.

Theorem 2.1.19. [1] Let S be a semigroup. Then the following are hold.

(1) Every left ideal is a bi-interior ideal of S.

(2) Every right ideal is a bi-interior ideal of S.

(3) Every ideal is a bi-interior ideal of S.

(4) Every quasi-ideal is a bi-interior ideal of S.

(5) Every interior ideal is a bi-interior ideal of S.

(6) If L is a bi-interior ideal of S then $LL \subseteq L$.

(7) The intersection of a right ideal and a left ideal of S is a bi-interior ideal of S.

(8) If B is a bi-interior ideal of S, then BS and SB are bi-interior ideals of S.

(9) If A and B are bi-interior ideals of S, then $A \cap B$ is a bi-interior ideal of S, if $A \cap B \neq \phi$.

(10) If B is a bi-interior ideal and T is a subsemigroup of S, then $B \cap T$ is a bi-interior ideal of S, if $B \cap T \neq \phi$.

Theorem 2.1.20. [1] Let S be a simple semigroup. Every bi-interior ideal is a bi-ideal of S.

Theorem 2.1.21. [1] Let S be a regular semigroup. Every bi-interior ideal is an ideal of S.

Theorem 2.1.22. [1] If L is a minimal left ideal and R is a minimal right ideal of a semigroup S, then $B = RL$ is a minimal bi-interior ideal of S.

Theorem 2.1.23. [1] Let A and C be subsemigroups of a semigroup S and $B = AC$. If A is a left ideal of S, then B is a bi-interior ideal of S.

2.2 Hypersemigroups

The fundamental definitions of the theory of hypersemigroups, which will be employed throughout the study, are recalled. Given the wide range of directions for the concept of hypersemigroups (semihypergroups, multisemigroups), in this work, we use the one discussed by Kehayopulu (see [2, 3, 4, 10]).

A *hypergroupoid* is a nonempty set H with a *hyperoperation* $\cdot$ by a mapping ;

$$\cdot : H \times H \to \mathcal{P}^*(H) \mid (a, b) \to a \cdot b$$

and operation

$$* : \mathcal{P}^*(H) \times \mathcal{P}^*(H) \to \mathcal{P}^*(H) \mid (A, B) \to A * B$$

defined by

$$A * B = \bigcup_{a \in A, b \in B} (a \cdot b)$$

for every $A, B \in \mathcal{P}^*(H)$ where $\mathcal{P}^*(H)$ is the set of all nonempty subsets of H. The structure $(H, \cdot)$ we denote by H.

Definition 2.2.1. [2] A hypergroupoid $(H, \cdot)$ is called a *hypersemigroup* if ;

$$\{x\} * (y \cdot z) = (x \cdot y) * \{z\}$$

for all $x, y, z \in H$.

Definition 2.2.2. [14] A hypergroupoid $(H;\cdot)$ is said to be a hypersemigroup if one of the following statements holds:

(1) $\{a\} * (b \cdot c) = (a \cdot b) * \{c\}$ for all $a, b, c \in H$;

(1) $\{a\} * (\{b\} * \{c\}) = (\{a\} * \{b\}) * \{c\}$ for all $a, b, c \in H$;

Lemma 2.2.3. [2] If H is a hypergroupoid and $A, B \in \mathcal{P}^*(H)$, then

1. $x \in A * B$ if and only if $x \in a \cdot b$, for some $a \in A, b \in B$.
2. If $a \in A$ and $b \in B$, then $a \cdot b \subseteq A * B$.

Lemma 2.2.4. [2] Let H be a hypergroupoid and $A, B, C, D \in \mathcal{P}^*(H)$. Then the following properties are hold:

1. if $A \subseteq B$ and $C \subseteq D$ then $A * C \subseteq B * D$;
2. if $A \subseteq B$ then $A * C \subseteq B * C$ and $C * A \subseteq C * B$;
3. $H * H \subseteq H$;
4. $H * A \subseteq H$ and $A * H \subseteq H$.

Lemma 2.2.5. [2] If H is a hypersemigroup, then the operation $*$ on $\mathcal{P}^*(H)$ is associative, that is $(A * B) * C = A * (B * C)$ for any $A, B, C \in \mathcal{P}^*(H)$.

Definition 2.2.6. [8] A non-empty subset B of the hypersemigroup H is called a $subhypersemigroup$ of H if $B * B \subseteq B$.

Definition 2.2.7. [7] A hyperideal I of a hypergroupoid H is called $proper$ if $A \neq H$. A hypersemigroup is called $simple$ if it has no proper hyperideal.

Proposition 2.2.8. [7] A hypersemigroup H is $simple$ if and only if $H = H * A * H$ for every $A \in \mathcal{P}^*(H)$.

Lemma 2.2.9. [3] A hypersemigroup $(H, \cdot)$ is $regular$ if and only if for any non-empty subset A of H, we have $A \subseteq A * H * A$.

Lemma 2.2.10. [4] If $(H, \cdot)$ is a regular hypersemigroup, then $S * S = S$.

Definition 2.2.11. [10] If $(H, \cdot)$ is an hypersemigroup, an element e of H is called $identity\ element$ if $a \cdot e = e \cdot a = \{a\}$ for every $a \in H$.

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Definition 2.2.12. [5] A non-empty subset A of a hypersemigroup H is called a $left$ (resp., $right$) hyperideal of H if $H * A \subseteq A$ (resp., $A * H \subseteq A$). If A is both a left and a right hyperideal of H, then it is called a $two-side\ hyperideal$ of H.

Remark 2.2.13. Two-side hyperideal of H is simply to called a $hyperideal$ of H.

Definition 2.2.14. [2] Let H be an hypersemigroup. A non-empty subset B of H is called $bi-hyperideal$ of H if

$$B * H * B \subseteq B.$$

Theorem 2.2.15. [3] Let H be a hypersemigroup. Then, the following conditions are equivalent.

1. H is regular.
2. $R \cap L \subseteq R * L$ for every right hyperideal R and every left hyperideal L of H.

Definition 2.2.16. [23] Let H be an hypersemigroup. A non-empty subset I of H is called $interior\ hyperideal$ of H if

$$H * I * H \subseteq I.$$

Definition 2.2.17. [2] Let H be a hypergroupoid. A non-empty subset Q of H is called a $quasi-hyperideal$ of H if

$$(Q * H) \cap (H * Q) \subseteq Q.$$

Definition 2.2.18. [25] Let H be a hypersemigroup. A left hyperideal L of H is called $minimal$ if for every left hyperideal A of H such that $A \subseteq L$, we have $A = L$.

Definition 2.2.19. [25] Let H be a hypersemigroup. A right hyperideal R of H is called $minimal$ if for every right hyperideal A of H such that $A \subseteq R$, we have $A = R$.

Definition 2.2.20. [25] Let H be a hypersemigroup. A bi-hyperideal B of H is called $minimal$ if for every bi-hyperideal A of H such that $A \subseteq B$, we have $A = B$.

CHAPTER 3

BI-INTERIOR HYPERIDEALS ON A HYPERSEMIGROUP

In this section, we introduce the concept of bi-interior hyperideals and study some its basic properties.

3.1 Bi-interior hyperideals

M.M.K. Rao [1] introduced the notion of bi-interior ideals of semigroups. In this subsection, we introduce the definition of bi-interior hyperideals on hypersemigroups.

Definition 3.1.1. Let H be a hypersemigroup. A non-empty subset A of H is said to be a $bi - interior\ hyperideal$ of H if A is a subhypersemigroup of H and $(H * A * H) \cap (A * H * A) \subseteq A$.

Example 3.1.2. [8] Let $H = \{a, b, c, d\}$ together with a hyperoperation $\circ$ defined by the following table:

$\circ$	a	b	c	d
a	a	b	c	d
b	a	a	a	a
c	a	a	$\{a, b\}$	a
d	a	a	$\{a, b\}$	$\{a, b\}$

It is easy to verify that, $(H, \circ)$ is a hypersemigroup. Let $B = \{a, b\}$. It is easy to see that B is a subhypersemigroup. We get

(1) $H * B = \bigcup\limits_{h \in H, b \in B} (h \circ b) = \{a, b\} = B$,

(2) $B * H = \bigcup\limits_{b \in B, h \in H} (b \circ h) = \{a, b, c, d\} = H$.

We have that $(H * B * H) \cap (B * H * B) = (B * H) \cap (H * B) \subseteq H \cap B = B$. This means that B is a bi-interior hyperideal of H.

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Lemma 3.1.3. Let A and B be two bi-interior hyperideals of a hypersemigroup H. If $A \cap B \neq \phi$, then $A \cap B$ is a bi-interior hyperideal of H.

Proof. Let A and B be two bi-interior hyperideals of H and $A \cap B \neq \phi$. First, We see that

$$(A \cap B) * (A \cap B) \subseteq A * A \subseteq A \text{ and } (A \cap B) * (A \cap B) \subseteq B * B \subseteq B$$

Thus, $(A \cap B) * (A \cap B) \subseteq A \cap B$, that is $A \cap B$ is a subhypersemigroup of H. Next, we shows that $[H * (A \cap B) * H] \cap [(A \cap B) * H * (A \cap B)] \subseteq (A \cap B)$. Consider

$$[H * (A \cap B) * H] \cap [(A \cap B) * H * (A \cap B)] \subseteq (H * A * H) \cap (A * H * A) \subseteq A,$$

and

$$[H * (A \cap B) * H] \cap [(A \cap B) * H * (A \cap B)] \subseteq (H * B * H) \cap (B * H * B) \subseteq B.$$

This means that $[H * (A \cap B) * H] \cap [(A \cap B) * H * (A \cap B)] \subseteq (A \cap B)$. Therefore, $A \cap B$ is a bi-interior hyperideal of H. □

Theorem 3.1.4. Let $\{B_i \mid i \in I\}$ be a family of all bi-interior hyperideals of H. If $\bigcap\{B_i \mid i \in I\} \neq \phi$, then $\bigcap\{B_i \mid i \in I\}$ is a bi-interior hyperideal of H.

Proof. Let $\{B_i \mid i \in I\}$ be a family of all bi-interior hyperideals of H and $\bigcap\{B_i \mid i \in I\} \neq \phi$. First, we shows that $\bigcap\{B_i \mid i \in I\}$ is a subhypersemigroup of H, we obtain $\bigcap\{B_i \mid i \in I\} * \bigcap\{B_i \mid i \in I\} \subseteq B_i * B_i \subseteq B_i$ for all $i \in I$, we prove $\bigcap\{B_i \mid i \in I\}$ is a subhypersemigroup of H. Next, we have that $[H * (\bigcap\{B_i \mid i \in I\}) * H] \cap [(\bigcap\{B_i \mid i \in I\}) * H * (\bigcap\{B_i \mid i \in I\})] \subseteq (H * B_i * H) \cap (B_i * H * B_i) \subseteq B_i$, for all $i \in I$. This mean that

$$[H * (\bigcap\{B_i \mid i \in I\}) * H] \cap [(\bigcap\{B_i \mid i \in I\}) * H * (\bigcap\{B_i \mid i \in I\})] \subseteq (\bigcap\{B_i \mid i \in I\}).$$

Therefore $\bigcap\{B_i \mid i \in I\}$ is a bi-interior hyperideal of H. □

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However, the following table shows that this is not the case for unions.

Example 3.1.5. [8] Let $H = \{a, b, c, d, f\}$ with a hyperoperation $\circ$ defined by the following table:

$\circ$	a	b	c	d	f
a	a	a	a	a	a
b	a	$\{a, b\}$	a	$\{a, d\}$	a
c	a	$\{a, f\}$	$\{a, c\}$	$\{a, c\}$	$\{a, f\}$
d	a	$\{a, b\}$	$\{a, d\}$	$\{a, d\}$	$\{a, b\}$
f	a	$\{a, f\}$	a	$\{a, c\}$	a

By the above table, we have that $B = \{a, b, d\}$ and $C = \{a, c\}$ are bi-interior hyperideals of H. We see that $B \cap C = \{a\}$ is a bi-interior hyperideal of H but $B \cup C = \{a, b, c, d\}$ is not a bi-interior hyperideal of H. Indeed, we see that $H * (B \cup C) = H$ and $(B \cup C) * H = H$, thus $[H * (B \cup C) * H] \cap [(B \cup C) * H * (B \cup C)] = H \cap H = H \nsubseteq B \cup C$.

Let H be a hypersemigroup. For any non-empty subset A of H. By Theorem 3.1.4., we get that any intersection of bi-interior hyperideal of H if it is not empty, it also a bi-interior hyperideal of H. A bi-interior hyperideal of H containing A defined by

$$\langle A \rangle_H := \bigcap \{B_i \mid B_i \text{ is a bi-interior hyperideal of } H \text{ and } A \subseteq B_i, \text{for all } i \in I\}$$

is called $bi - interior\ hyperideal\ of\ H\ generated\ by\ A$.

Remark. We will denote $\langle A \rangle_H$ by $(A)_{bn}$.

Let H be a hypersemigroup and A is a non-empty subset of H. We denoted by

$(A)_l$ the smallest left hyperideal of H containing A;

$(A)_r$ the smallest right hyperideal of H containing A;

$(A)_j$ the smallest two-side hyperideal of H containing A;

$(A)_q$ the smallest quasi-hyperideal of H containing A;

$(A)_b$ the smallest bi-hyperideal of H containing A;

$(A)_i$ the smallest interior hyperideal of H containing A.

They were illustrated in [2,26] that

1. $(A)_l = A \cup H * A$;
2. $(A)_r = A \cup A * H$;
3. $(A)_j = A \cup (H * A) \cup (A * H) \cup (H * A * H)$;
4. $(A)_q = A \cup [(A * H) \cap (H * A)]$;
5. $(A)_b = A \cup A^2 \cup (A * H * A)$;
6. $(A)_i = A \cup A^2 \cup (H * A * H)$.

We now introduce a new class of hyperideals that plays a crucial role in this thesis. Next theorem, we will illustrate the construction of $(A)_{bn}$.

Theorem 3.1.6. Let A be a non-empty subset of a hypersemigroup H. Then

$$(A)_{bn} = A \cup A^2 \cup [(A * H * A) \cap (H * A * H)].$$

Proof. Let $B = A \cup A^2 \cup [(A * H * A) \cap (H * A * H)]$. We show that B is the smallest bi-interior hyperideal of H containing A. Consider

$$\begin{aligned} B * B &\subseteq A^2 \cup A^3 \cup A^4 \cup (A * H * A) \\ &\subseteq A^2 \cup (A * H * A) \cup (A * H * A) \cup (A * H * A) \\ &= A^2 \cup (A * H * A) \end{aligned}$$

and

$$\begin{aligned} B * B &\subseteq A^2 \cup A^3 \cup A^4 \cup (H * A * H) \\ &\subseteq A^2 \cup (H * A * H) \cup (H * A * H) \cup (H * A * H) \\ &= A^2 \cup (H * A * H) \end{aligned}$$

Thus, we have

$$\begin{aligned} B * B &\subseteq [A^2 \cup (A * H * A)] \cap [A^2 \cup (H * A * H)] \\ &= A^2 \cup [(A * H * A) \cap (H * A * H)] \\ &\subseteq A \cup A^2 \cup [(A * H * A) \cap (H * A * H)] \\ &= B. \end{aligned}$$

This shows that B is a subhypersemigroup of H. Consider

$$B * H * B \subseteq (A \cup A^2 \cup (A * H * A)) * H * (A \cup A^2 \cup (A * H * A)) \subseteq A * H * A$$

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and

$$B * H * B \subseteq (A \cup A^2 \cup (H * A * H)) * H * (A \cup A^2 \cup (H * A * H)) \subseteq H * A * H.$$

Thus we have $B * H * B \subseteq (A * H * A) \cap (H * A * H) \subseteq B$. These illustrate that B is a bi-interior hyperideal of H.

Now, let C be a bi-interior hyperideal of H such that $A \subseteq C$. Then

$$\begin{aligned} B &= A \cup A^2 \cup [(A * H * A) \cap (H * A * H)] \\ &\subseteq C \cup C^2 \cup [(C * H * C) \cup (H * C * H)] \\ &\subseteq C \cup C \cup C \\ &= C. \end{aligned}$$

This mean that B is the smallest bi-interior hyperideal of H.

By the definition of $(A)_{bn}$ and the smallest property of B, we obtain $(A)_{bn} \subseteq B \subseteq (A)_{bn}$. Therefore, we have $(A)_{bn} = A \cup A^2 \cup [(A * H * A) \cap (H * A * H)]$. □

3.2 Subbi-interior hyperideals

In this subsection, we introduce the concept of subbi-interior hyperideals and some properties of such subbi-interior hyperideals are studied.

Definition 3.2.1. Let B be a bi-interior hyperideal of H. A non-empty subset S of B is called $subbi - interior\ hyperideal$ of B if S is a bi-interior hyperideal of B.

Example 3.2.2. From the example 3.1.5., it easy to verify that $J = \{a, b, f\}$ and $K = \{a, c, d\}$ are bi-interior hyperideals of H. Let $M = \{a, b\}$ be a subset of J and $N = \{a, d\}$ is a subset of K. Consider

$$\begin{aligned} [H * M * H] \cap [M * H * M] &= [\{a, b, d\} * \{a, b\}] \cap [\{a, b, f\} * H] \\ &= \{a, b\} \cap H \\ &= \{a, b\} \\ &\subseteq M. \end{aligned}$$

and

$$\begin{aligned}[J * M * J] \cap [M * J * M] &= (J * J) \cap (M * M)\\ &\subseteq J \cap M\\ &= M.\end{aligned}$$

Similarly,

$$\begin{aligned}[H * N * H] \cap [N * H * N] &= [\{a, c, d\} * H] \cap [\{a, b, d\} * N]\\ &\subseteq H \cap N\\ &= N\end{aligned}$$

and

$$\begin{aligned}[K * N * K] \cap [N * K * N] &= (K * K) \cap (N * N)\\ &\subseteq K \cap N\\ &= N.\end{aligned}$$

Hence M is a subbi-interior hyperideal of bi-interior hyperideal J of H and N is a subbi-interior hyperideal of bi-interior hyperideal K of H. Moreover, $M \cap N = \{a\}$ is also a subbi-interior hyperideal of $J \cap K$ of H.

Lemma 3.2.3. Let A and B be bi-interior hyperideals of H and S be a subbi-interior hyperideal of A and T be a subbi-interior hyperideal of B. If $A \cap B \neq \phi$ and $S \cap T \neq \phi$, then $S \cap T$ is a subbi-interior hyperideal of $A \cap B$.

Proof. Let S be a subbi-interior hyperideal of a bi-interior hyperideal A of H and T be a subbi-interior hyperideal of a bi-interior hyperideal B of H where $A \cap B \neq \phi$ and $S \cap T \neq \phi$. By Lemma 3.1.3, we obtain $S \cap T$ is a bi-interior hyperideal of H. Since S is a bi-interior hyperideal of A and T is a bi-interior hyperideal of B, so

$$\begin{aligned}&[(S \cap T) * (A \cap B) * (S \cap T)] \cap [[(A \cap B) * (S \cap T) * (A \cap B)]\\ \subseteq\ &[S * (A \cap B) * S] \cap [(A \cap B) * S * (A \cap B)]\\ \subseteq\ &[S * A * S] \cap [A * S * A]\\ \subseteq\ &S\end{aligned}$$

and

$$
\begin{aligned}
& [(S\cap T)*(A\cap B)*(S\cap T)]\cap[(A\cap B)*(S\cap T)*(A\cap B)] \\
\subseteq\ & [T*(A\cap B)*T]\cap[(A\cap B)*T*(A\cap B)] \\
\subseteq\ & [T*B*T]\cap[B*T*B] \\
\subseteq\ & T
\end{aligned}
$$

This implies that $[(S\cap T)*(A\cap B)*(S\cap T)]\cap[(A\cap B)*(S\cap T)*(A\cap B)]\subseteq S*T$. Therefore $S\cap T$ is a subbi-interior hyperideal of $A\cap B$. $\square$

Theorem 3.2.4. Let T_i be a subbi-interior hyperideal of B_i, for all $i\in I$ when I is an index set. If $\bigcap\limits_{i\in I} B_i\neq\phi$ and $\bigcap\limits_{i\in I} T_i\neq\phi$, then $\bigcap\limits_{i\in I} T_i$ is a subbi-interior hyperideal of $\bigcap\limits_{i\in I} B_i$ for all $i\in I$.

Proof. Since T_i is a subbi-interior hyperideal of B_i and $T_i\neq\phi$, for all $i\in I$. Then T_i is a bi-interior hyperideal of B_i for all $i\in I$. By Lemma 3.2.3, $\bigcap\limits_{i\in I} T_i$ is a bi-interior hyperideal of H and $\bigcap\limits_{i\in I} T_i$ is a bi-interior hyperideal of $\bigcap\limits_{i\in I} B_i$ for all $i\in I$. Therefore $\bigcap\limits_{i\in I} T_i$ is a subbi-interior hyperideal of $\bigcap\limits_{i\in I} T_i$. $\square$

3.3 Cartesian product and direct product on bi-interior hyperideals

In 1993, P. Corsini [24] introduced a direct product of two hypersemigroups. It has been proved that the Cartesian product of these two hypersemigroups is a hypersemigroup.

In this subsection, we introduce the notion of the direct product of bi-interior hyperideals and study some properties of such direct product of bi-interior hyperideals on hypersemigroups.

First, we present the definition of direct product of two hypersemigroups as follow:

Definition 3.3.1. Let $(H;\cdot_H)$ and $(K;\cdot_K)$ be two hypersemigroups. The *Direct product* of $(H;\cdot_H)$ and $(K;\cdot_K)$ denoted by $H\times K$, and define a hyperoperation

$$\circ_d:(H\times K)\times(H\times K)\to\mathcal{P}^*(H\times K)$$

by

$$(a,b)\circ_d(c,d):=(a\cdot_H c)\times(b\cdot_K d).$$

For all $a, c \in H$ and $b, d \in K$.

Remark. (1) The binary operation $*_d : \mathcal{P}^*(H \times K) \times \mathcal{P}^*(H \times K) \to \mathcal{P}^*(H \times K)$ on $\mathcal{P}^*(H \times K)$ defined by;

$$(A \times B) *_d (C \times D) := (A *_H C) \times (B *_K D).$$

For any $A \times B, C \times D \in \mathcal{P}^*(H \times K)$ such that $*_H$ is a operation on $\mathcal{P}^*(H)$ and $*_K$ is a operation on $\mathcal{P}^*(K)$.

(2) Let $(a, b), (c, d), (e, f) \in H \times K$, then we have that;

$$\{(a, b)\} *_d [(c, d) \circ_d (e, f)] := [\{a\} *_H (c \cdot_H e)] \times [\{b\} *_K (d \cdot_K f)].$$

and

$$[(a, b) \circ_d [(c, d)] *_d \{(e, f)\} := [(a \cdot_H c) *_H \{e\}] \times [(b \cdot_K d) *_K \{f\}].$$

Example 3.3.2. [8] Let $H = \{1, 2, 3\}$ be an hypersemigroups defined by ;

$\cdot_H$	1	2	3
1	1	1	1
2	1	$\{1,2\}$	1
3	1	$\{1,2\}$	$\{1,2\}$

$K = \{a, b, c, d\}$ is an hypersemigroups defined by ;

$\cdot_K$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	a
b	a	$\{a,b\}$	$\{a,c\}$	a
c	a	$\{a,b\}$	$\{a,c\}$	a
d	a	$\{a,b\}$	$\{a,c\}$	a

For the above tables, we obtain that $H \times K = \bigcup\limits_{x \in H, y \in K} (x \cdot_H y) \circ_d \bigcup\limits_{x' \in H, y' \in K} (x' \cdot_K y') = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

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Lemma 3.3.3. Let H and K be two hypersemigroups. We obtain $H \times K$ is a hypersemigroup.

Proof. Let $(a,b),(c,d),(e,f) \in H \times K$. Consider

$$\begin{aligned}
\{(a,b)\} *_d [(c,d) \circ_d (e,f)] &= \{(a,b)\} *_d [(c \cdot_H e) \times (d \cdot_K f)] \\
&= [\{a\} *_H (c \cdot_H e)] \times [\{b\} *_H (d \cdot_H f)] \\
&= [(a \cdot_H c) *_H \{e\}] \times [(b \cdot_K d) *_K \{f\}] \\
&= [(a \cdot_H c) \times (b \cdot_K d)] *_d \{e,f\}] \\
&= [(a,b) \circ_d (c,d)] *_d \{e,f\}].
\end{aligned}$$

Therefore $H \times K$ is a hypersemigroup. □

Proposition 3.3.4. Let A be a subhypersemigroup of a hypersemigroup H and B be a subhypersemigroup of a hypersemigroup K. Then $A \times B$ is a subhypersemigroup of $H \times K$.

Proof. Let A and B be subhypersemigroups of H and K, respectively. By Lemma 3.3.3., we have that $A \times B$ is a hypersemigroup. Next, let $(a,b),(c,d) \in A \times B$. Then $(a,b) \circ_d (c,d) = (a \cdot_H c) \times (b \cdot_K d) \subseteq A \times B$. Therefore $A \times B$ is a subhypersemigroup of $H \times K$. □

Theorem 3.3.5. Let B be a bi-interior hyperideal of H and C be a bi-interior hyperideal of K. Then $B \times C$ is a bi-interior hyperideal of $H \times K$.

Proof. Let B be a bi-interior hyperideal of H and C be a bi-interior hyperideal of K. By Proposition 3.3.4, we obtain $B \times C$ is a subhypersemigroup of $H \times K$. Next, consider

$$\begin{aligned}
& [(B \times C) *_d (H \times K) *_d (B \times C)] \cap [(H \times K) *_d (B \times C) *_d (H \times K)] \\
= \ & [(B *_H H) \times (C *_K K)] *_d (B \times C) \cap [(H *_H B) \times (K *_K C)] *_d (H \times K) \\
= \ & [(B *_H H *_H B) \times (C *_K K *_K C)] \cap [(H *_H B *_H H) \times (K *_K C *_K K)] \\
= \ & [(B *_H H *_H B) \cap (H *_H B *_H H)] \times [(C *_K K *_K C) \cap (K *_K C *_K K)] \\
\subseteq \ & B \times C.
\end{aligned}$$

Therefore $B \times C$ is a bi-interior hyperideal of $H \times K$. □

By the above lemma, we illustrated the generalized of a direct product of a family of hypersemigroups, subhypersemigroups and bi-interior hyperideals.

Definition 3.3.6. Let $\{H_i \mid i \in I\}$ be a family of hypersemigroups. The Cartesian product $\prod\limits_{i\in I} H_i$ is defined by;

$$\prod_{i\in I} H_i := \{f : I \to (\bigcup_{i\in I} H_i \times \bigcup_{i\in I} H_i) \mid f(i) \in H_i \times H_i; \text{ for all } i \in I\}.$$

The direct product of $\{H_i \mid i \in I\}$ is the Cartesian product $\mathcal{H} = \prod\limits_{i\in I} H_i$ with the hyper operation $\circ_d : \mathcal{H} \times \mathcal{H} \to \mathcal{P}^*(\mathcal{H})$, for which the product of two functions $f, g \in \prod\limits_{i\in I} H_i$ is the function $f \circ_d g : (\bigcup\limits_{i\in I} H_i \times \bigcup\limits_{i\in I} H_i) \times (\bigcup\limits_{i\in I} H_i \times \bigcup\limits_{i\in I} H_i) \to \mathcal{P}^*(\bigcup\limits_{i\in I} H_i \times \bigcup\limits_{i\in I} H_i)$ defined by

$$(f \circ_d g)(i) \;=\; f(i) \circ_d g(i) \qquad \text{for all } \; i \in I.$$

Lemma 3.3.7. Let $\{H_i \mid i \in I\}$ be a family of hypersemigroups. We obtain $\prod\limits_{i\in I} H_i$ is a hypersemigroup.

Proof. Let $f, g \in \prod\limits_{i\in I} H_i$. Then $f(i), g(i), h(i) \in H_i \times H_i$; for all $i \in I$, we have that $f(i) = (\{a_i\}, \{b_i\})$, $g(i) = (a_i', b_i')$ and $h(i) = (a_i'', b_i'')$. Consider

$$\begin{aligned}
f(i) *_{d_i} ((g \circ_{d_i} h)(i)) &= (\{a_i\}, \{b_i\}) *_{d_i} [(a_i', b_i') \circ_{d_i} (a_i'', b_i'')] \\
&= (\{a_i\}, \{b_i\}) *_{d_i} [(a_i' \circ_{H_i} a_i'') \times (b_i' \circ_{H_i} b_i'')] \\
&= [\{a_i\} *_{H_i} (a_i' \circ_{H_i} a_i'')] \times [\{b_i\} *_{H_i} (b_i' \circ_{H_i} b_i'')] \\
&= [(a_i \circ_{H_i} a_i') *_{H_i} \{a_i''\}] \times [(b_i \circ_{H_i} b_i') *_{H_i} \{b_i''\}] \\
&= [(a_i \circ_{H_i} a_i') \times (b_i \circ_{H_i} b_i')] *_{d_i} (\{a_i''\}, \{b_i''\}) \\
&= [(a_i, b_i) \circ_{d_i} (a_i', b_i')] *_{d_i} (\{a_i''\}, \{b_i''\}) \\
&= (f(i) \circ_{d_i} g(i)) *_{d_i} h(i) \\
&= (f \circ_{d_i} g)(i) *_{d_i} h(i)
\end{aligned}$$

Then $f(i) *_{d_i} ((g \circ_{d_i} h)(i)) = (f \circ_{d_i} g)(i) *_{d_i} h(i)$ for all $i \in I$. Hence $\prod\limits_{i\in I} H_i$ is a hypersemigroup. □

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Proposition 3.3.8. Let $\{S_i \mid i \in I\}$ be a family of subhypersemigroups of $\{H_i \mid i \in I\}$, for all $i \in I$. Then $\prod\limits_{i\in I} S_i$ is a subhypersemigroup of $\prod\limits_{i\in I} H_i$.

Proof. Let $\{S_i \mid i \in I\}$ be a family of subhypersemigroup $\{H_i \mid i \in I\}$, for all $i \in I$. By Lemma 3.3.7., then $\prod\limits_{i\in I} S_i$ is a hypersemigroup. Next, let $f, g \in \prod\limits_{i\in I} S_i$. By Proposition 3.3.4., then we obtain $f(i) = (a_i, b_i)$, $g(i) = (a'_i, b'_i)$ for all $i \in I$. Consider

$$\begin{aligned} f(i) \circ_{d_i} g(i) &= (a_i, b_i) \circ_{d_i} (a'_i, b'_i) \\ &= (a_i \cdot_{S_i} a'_i) \times z(b_i \cdot_{S_i} b'_i) \\ &\subseteq S_i \times S_i. \end{aligned}$$

This implies that $f(i) \circ_{d_i} g(i) \subseteq S_i \times S_i$ for all $i \in I$. Therefore $\prod\limits_{i\in I} S_i$ is a subhypersemigroup of $\prod\limits_{i\in I} H_i$. □

Theorem 3.3.9. Let $\{B_i \mid i \in I\}$ be a family of bi-interior hyperideals of $\{H_i, i \in I\}$. Then $\prod\limits_{i\in I} B_i$ is a bi-interior hyperideal of $\prod\limits_{i\in I} H_i$.

Proof. Let $\{B_i \mid i \in I\}$ be a family of bi-interior hyperideals of $\{H_i, i \in I\}$. By proposition 3.3.8., then $\prod\limits_{i\in I} B_i$ is a subhypersemigroups of $\prod\limits_{i\in I} H_i$. Since B_i is a bi-interior hyperideal of H_i for all $i \in I$, so $(H_i *_{d_i} B_i *_{d_i} H_i) \cap (B_i *_{d_i} H_i *_{d_i} B_i) \subseteq B_i$ for all $i \in I$. That is $(\prod\limits_{i\in I} B_i *_d \prod\limits_{i\in I} H_i *_d \prod\limits_{i\in I} B_i) \bigcap (\prod\limits_{i\in I} H_i *_d \prod\limits_{i\in I} B_i *_d \prod\limits_{i\in I} H_i) \subseteq \prod\limits_{i\in I} B_i$. Therefore $\prod\limits_{i\in I} B_i$ is a bi-interior hyperideal of $\prod\limits_{i\in I} H_i$. □

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CHAPTER 4

THE RELATIONS BETWEEN BI-INTERIOR HYPERIDEALS AND OTHERS HYPERIDEALS ON HYPERSEMIGROUPS

In this section, we examine relations between bi-interior hyperideals and others hyperideals on hypersemigroups. Also, we studied their relations on simple and on regular hypersemigroups.

First, we present the closed relations between bi-interior hyperideals and left, right, two-side, quasi-, bi- and interior hyperideals in hypersemigroups.

4.1 The relations between bi-interior hyperideals and others hyperideals on hypersemigroups

Proposition 4.1.1. Let H be a hypersemigroup. Then the following statements hold.

(1) Every left hyperideal of H is a bi-interior hyperideal of H.

(2) Every right hyperideal of H is a bi-interior hyperideal of H.

(3) Every two-side hyperideal of H is a bi-interior hyperideal of H.

Proof. (1) Let L be a left hyperideal of H. We obtain $L * L \subseteq H * L \subseteq L$ and we have

$$[(H * L) * H] \cap [(L * H) * L] \subseteq L * H \cap H * L \subseteq H \cap L = L.$$

Therefore L is a bi-interior hyperideal of H.

(2) Let R be a right hyperideal of H. We obtain $R * R \subseteq R * H \subseteq R$ and we have

$$[H * (R * H)] \cap [(R * H) * R] \subseteq H * R \cap R * R \subseteq H \cap R = R.$$

Therefore R is a bi-interior hyperideal of H.

(3) Let I be a two-side hyperideal of H. Then I is both a left and a right hyperideal of H. By (1) and (2), we have that I is a bi-interior hyperideal of H. □

Lemma 4.1.2. The intersection of a right and a left hyperideal of H, if it is a non-empty set, then it is a bi-interior hyperideal of H.

Proof. Let R and L be a right and a left hyperideal of H, respectively. Suppose that $R \cap L \neq \phi$. Consider $(R \cap L) * (R \cap L) \subseteq R * H \subseteq R$ and $(R \cap L) * (R \cap L) \subseteq H * L \subseteq L$. Thus, we obtain $(R \cap L) * (R \cap L) \subseteq R \cap L$. Now, we see that

$$[H*(R\cap L)*H]\cap[(R\cap L)*H*(R\cap L)] \subseteq (R\cap L)*H*(R\cap L) \subseteq R*(H*H) \subseteq R*H \subseteq R.$$

and

$$[H*(R\cap L)*H]\cap[(R\cap L)*H*(R\cap L)] \subseteq (R\cap L)*H*(R\cap L) \subseteq (H*H)*L \subseteq H*L \subseteq L.$$

This implies that $[H * (R \cap L) * H] \cap [(R \cap L) * H * (R \cap L)] \subseteq R \cap L$. Therefore $R \cap L$ is a bi-interior hyperideal of H. □

Theorem 4.1.3. Let H be a hypersemigroup. Then every quasi-hyperideal of H is a bi-interior hyperideal of H.

Proof. Let A be a quasi-hyperideal of H. Since $A * A \subseteq A * H$ and $A * A \subseteq H * A$. This implies that $A * A \subseteq (A * H) \cap (H * A) \subseteq A$. Consider

$$(H * A * H) \cap (A * H * A) \subseteq A * H * A \subseteq A * H,$$

and

$$(H * A * H) \cap (A * H * A) \subseteq A * H * A \subseteq H * A,$$

we obtain

$$(H * A * H) \cap (A * H * A) \subseteq (A * H) \cap (H * A) \subseteq A.$$

Therefore A is a bi-interior hyperideal of H. □

Theorem 4.1.4. Let H be a hypersemigroup. Then every bi-hyperideal of H is a bi-interior hyperideal of H.

Proof. Let B be a bi-hyperideal of H, then B is a subhypersemigroup of H. By hypothesis, we have that $B * H * B \subseteq B$. Then $(H * B * H) \cap (B * H * B) \subseteq B * H * B \subseteq B$. Therefore B is a bi-interior hyperideal of H. □

Theorem 4.1.5. Let H be a hypersemigroup. Then every interior hyperideal of H is a bi-interior hyperideal of H.

Proof. Let B be an interior hyperideal of H. Then $H * B * H \subseteq B$. Consider

$$(H * B * H) \cap (B * H * B) \subseteq B \cap (B * H * B) \subseteq B \cap H = B.$$

Therefore B is a bi-interior hyperideal of H. $\square$

Lemma 4.1.6. Let H be a hypersemigroup with the identity e. Then,

$$A * \{e\} = A = \{e\} * A.$$

for all non-empty subset A of H.

Proof. Let A be a non-empty subset of H. By Definition 2.2.6, we have that

$$A * \{e\} = \bigcup_{a \in A} (a \circ e) = \bigcup_{a \in A} \{a\} = A$$

and

$$\{e\} * A = \bigcup_{a \in A} (e \circ a) = \bigcup_{a \in A} \{a\} = A.$$

$\square$

Theorem 4.1.7. Let H be a hypersemigroup with identity e. Suppose that $H = H * \{a\}$ for all $a \in H$. Then every bi-interior hyperideal of H is a quasi-hyperideal of H.

Proof. Let B be a bi-interior hyperideal of H. By hypothesis, we have that $H = H * A$ for any non-empty subset A of H. This implies that $H = H * B$. By Lemma 4.1.6., we obtain $H*B = H*B*\{e\} \subseteq H*B*H$. Then $(H*B) \cap (B*H) \subseteq (H*B*H) \cap (B*H*B) \subseteq B$. Therefore B is a quasi-hyperideal of H. $\square$

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Theorem 4.1.8. Let H be a hypersemigroup with the identity e. Then every bi-interior hyperideal of H is an interior hyperideal of H.

Proof. Let B be a bi-interior hyperideal of H. By Lemma 4.1.6., we have that $B = \{e\} * B * \{e\} \subseteq H * B * H$. Then

$$B * H * B \subseteq (H * B * H) * H * B \subseteq H * B * H.$$

Thus

$$B * H * B = (H * B * H) \cap (B * H * B) \subseteq B.$$

Therefore B is an interior hyperideal of H. □

4.2 The relations between bi-interior hyperideals and others hyperideals on simple hypersemigroups

The following results illustrates that the concept of bi-interior hyperideals in simple hypersemigroups.

Lemma 4.2.1. Let H be a hypersemigroup. Then the following statements hold.

(1) If H is left simple, then every bi-interior hyperideal of H is a right hyperideal of H.

(2) If H is right simple, then every bi-interior hyperideal of H is a left hyperideal of H.

(3) If H is simple, then every bi-interior hyperideal of H is a two-side hyperideal of H.

Proof. (1) Let A be a bi-interior hyperideal of a left simple hypersemigroup H. We obtain that $H * A$ is a left hyperideal of H. By hypothesis, we have that $H * A = H$ and also have $A * H = A * (H * A) = A * H * A$. Then, we have

$$A * H = A * H * A = A * (H * A) * A \subseteq H * A * H.$$

Thus, we obtain $A * H \subseteq (A * H * A) \cap (H * A * H) \subseteq A$. Therefore A is a right hyperideal of H.

(2) Let A be a bi-interior hyperideal of a right simple hypersemigroups H. We see that $A * H$ is a right hyperideal of H. By hypothesis, we have that $A * H = H$ and also have $H * A = (A * H) * A = A * H * A$. Then we have

$$H * A = (A * H) * A = A * (A * H) * A \subseteq H * A * H.$$

Thus, we obtain $H * A \subseteq (A * H * A) \cap (H * A * H) \subseteq A$. Therefore A is a left hyperideal of H.

(3) Let A be a bi-interior hyperideal of a simple hypersemigroup H. By (1) and (2), we obtain that A is a two-side hyperideal of H. □

Theorem 4.2.2. Let H be a simple hypersemigroup. Then every bi-interior hyperideal of H is a bi-hyperideal of H.

Proof. Let B be a bi-interior hyperideal of H. By Theorem 4.1.3.(3), we have that B is a two-side hyperideal of H. Since H is a simple hypersemigroup, we obtain $H = H * B * H$. Then

$$B * H * B = H \cap (B * H * B) = (H * B * H) \cap (B * H * B) \subseteq B.$$

Therefore B is a bi-hyperideal of H. □

4.3 The relations between bi-interior hyperideals and others hyperideals on regular hypersemigroups

The following gives a characterized bi-interior hyperideals in regular hypersemigroups.

Lemma 4.3.1. Let H be a regular hypersemigroup and B is a non-empty subset of H. Then we have $B \subseteq (H * B * H) \cap (B * H * B)$.

Proof. Let B be a non-empty subset of H. We obtain

$$B \subseteq B * H * B \text{ and } B \subseteq B * H * B \subseteq B * H * (B * H * B) \subseteq H * B * H.$$

Therefore $B \subseteq (H * B * H) \cap (B * H * B)$. □

Theorem 4.3.2. Let H be a regular hypersemigroup. Then the following statements are equivalent.

(1) B is a bi-interior hyperideal of H.

(2) $B = R * L$ for some right hyperideal R and left hyperideal L of H.

Proof. $(1) \Rightarrow (2)$. Let B be a bi-interior hyperideal of H, By Lemma 4.1.10, we have that

$$\begin{aligned} B * H * H * B &\subseteq [H * (B * H * H * B) * H] \cap [(B * H * H * B) * H * (B * H * H * B)] \\ &\subseteq [H * B * (H * H * B * H)] \cap [B * (H * H * B * H * B * H * H) * B] \\ &\subseteq (H * B * H) \cap (B * H * B). \end{aligned}$$

We consider

$$\begin{aligned} B &\subseteq B * H * B \\ &\subseteq (B * H) * (B * H * B) \qquad \text{(Since } H \text{ is regular)} \\ &\subseteq (B * H) * (H * B) \\ &\subseteq (H * B * H) \cap (B * H * B) \\ &\subseteq B. \end{aligned}$$

Thus $B = (B * H) * (H * B)$. Since $B * H$ and $H * B$ is a right and a left hyperideal of H, respectively. So we obtain (2).

$(2) \Rightarrow (1)$. We consider $B * B = (R * L) * (R * L) \subseteq R * L = B$ and

$$\begin{aligned} (B * H * B) \cap (H * B * H) &\subseteq B * H * B \\ &= (R * L) * H * (R * L) \\ &= R * (L * H * R * L) \\ &\subseteq R * L \\ &= B. \end{aligned}$$

Hence B is a bi-interior hyperideal of H. □

Theorem 4.3.3. Let H be a hypersemigroup. Then the following conditions are equivalent.

(1) H is regular.

(2) $A = (A * H * A) \cap (H * A * H)$ for every bi-interior hyperideal A of H.

(3) $A \cap L \subseteq A * L$ for every bi-interior hyperideal A and every left hyperideal L of H.

(4) $R \cap A \subseteq R * A$ for every right hyperideal R and every bi-interior hyperideal A of H.

Proof. $(1) \Rightarrow (2)$. Let A be a bi-interior hyperideal of H. By Lemma 4.1.10., we have

$$A \subseteq (A * H * A) \cap (H * A * H) \subseteq A.$$

This implies that $A = (A * H * A) \cap (H * A * H)$.

$(2) \Rightarrow (1)$. We prove this direction by using Theorem 4.1.2. Let R and L be a right and a left hyperideal of H, respectively. If $R \cap L = \phi$, by Theorem 4.1.2, we complete the proof. Suppose that $R \cap L \neq \phi$, by Lemma 4.1.2, $R \cap L$ is a bi-interior hyperideal of H. This implies that

$$\begin{aligned} R \cap L &= [(R \cap L) * H * (R \cap L)] \cap [H * (R \cap L) * H] \qquad \text{(by presumption)} \\ &\subseteq (R \cap L) * H * (R \cap L) \\ &\subseteq R * H * L \\ &\subseteq R * L. \end{aligned}$$

By Theorem 2.2.4, H is regular.

$(1) \Rightarrow (3)$. Let B and L be a bi-interior hyperideal and a left hyperideal of H, respectively. Since H is regular, we obtain

$$B \cap L \subseteq (B \cap L) * H * (B \cap L) \subseteq B * H * L \subseteq B * L.$$

$(3) \Rightarrow (1)$. Let A be a non-empty subset of H. Then,

$$\begin{aligned} A &\subseteq (A)_{bn} \cap (A)_l \\ &\subseteq (A)_{bn} * (A)_l \qquad \text{(by preconsumption)} \\ &= (A \cup A^2 \cup [(A * H * A) \cap (H * A * H)]) * [A \cup (H * A)] \\ &\subseteq (A \cup A^2 \cup (A * H * A)) * [A \cup (H * A)] \\ &\subseteq A^2 \cup (A * H * A). \end{aligned}$$

The proof is done if $A \subseteq A * H * A$. Suppose that $A \subseteq A^2$. Then $A \subseteq A * A \subseteq A * (A * A) \subseteq A * H * A$. Therefore H is regular.

$(1) \Leftrightarrow (4)$. The proof of this equivalence can be done similarly to $(1) \Leftrightarrow (3)$. □

We introduce the concept of bi-interior hyperideals is characterized by their corresponding left and right hyperideals.

Definition 4.3.4. Let H be a hypersemigroup. A bi-interior hyperideal B of H is called *minimal* if for every bi-interior hyperideal A of H such that $A \subseteq B$, we have $A = B$.

The following theorem illustrates that the product of any minimal right and left hyperideals is a minimal bi-interior hyperideal of a hypersemigroup.

Theorem 4.3.5. Let H be a hypersemigroup. Suppose that R and L is a minimal right and a minimal left hyperideal of H, respectively. We have that $R * L$ is a minimal bi-interior hyperideal of H.

Proof. Let R and L be a minimal right and a minimal left hyperideal of H, respectively. By the definition of the operation $*$ on $\mathcal{P}^*(H)$, it is clear that $R*L \neq \phi$. We let $A = R*L$. Consider

$$A * A = (R * L) * (R * L) \subseteq R * L = A.$$

and

$$A * H * A = (R * L) * H * (R * L) \subseteq R * H * L \subseteq R * L = A.$$

Thus A is a bi-hyperideal of H. By Theorem 4.1.6., we have that A is a bi-interior hyperideal of H. Now, we let B be a bi-interior hyperideal of H such that $B \subseteq A$. We obtain

$$H * (H * B) = (H * H) * B \subseteq H * B$$

and

$$(B * H) * H = B * (H * H) \subseteq B * H.$$

That is $H * B$ and $B * H$ is a left and a right hyperideal of H, respectively. Since R and L are minimal, we obtain $H * B = L$ and $B * H = R$. Then

$$A = R * L = (B * H) * (H * B) = B * (H * H) * B \subseteq B * H * B$$

and

$$A = R*L = R*(L*H) = (B*H)*[(H*B)*H] = (B*H*H)*B*H \subseteq H*B*H.$$

Thus $A \subseteq (H * B * H) \cap (B * H * B) \subseteq B$. This implies that $A = B$. Therefore A is a minimal bi-interior hyperideal of H. □

CHAPTER 5

CONCLUSIONS

The aim of this thesis is to introduce the notion of bi-interior hyperideals on hypersemigroups and we study some properties of such bi-interior hyperideals. Some properties are obtained. The results are follows:

1. Let H be a hypersemigroup. A non-empty subset A of H is said to be a $bi - interior\ hyperideal$ of H if A is a subhypersemigroup of H and $(H * A * H) \cap (A * H * A) \subseteq A$.

   From the 1., the following results are derived:

   1.1) Let A and B be bi-interior hyperideals of a hypersemigroup H. If $A \cap B \neq \phi$, then $A \cap B$ is a bi-interior hyperideal of H.

   1.2) Let $\{B_i \mid i \in I\}$ be a family of all bi-interior hyperideals of H. If $\bigcap\{B_i \mid i \in I\} \neq \phi$, then $\bigcap\{B_i \mid i \in I\}$ is a bi-interior hyperideal of H.

2. A non-empty subset S of the bi-interior hyperideal B of H is called $subbi - interior\ hyperideal$ of bi-interior hyperideal B of H if S is a bi-interior hyperideal of H and also S is a bi-inteior hyperideal of B.

   From the 2., the following results are derived:

   2.1) Let S be a subbi-interior hyperideal of bi-interior hyperideal A of H and T is a subbi-interior hyperideal of bi-interior hyperideal B of H. If $A \cap B \neq \phi$ and $S \cap T \neq \phi$, then $S \cap T$ is a subbi-interior hyperideal of a bi-interior hyperideal $A \cap B$ of H.

   2.2) Let T_i be a subbi-interior hyperideal of bi-interior hyperideal B_i of H for all $i \in I$ when I is an index set. If $\bigcap T_i \neq \phi$ then $\bigcap T_i$ is a subbi-interior hyperideal of H.

3. Let $(H;\cdot_H)$ and $(K;\cdot_K)$ be two hypersemigroups. The *Direct product* of $(H;\cdot_H)$ and $(K;\cdot_K)$ denoted by $H \times K$, and define a hyperoperation

$$\circ_d : (H \times K) \times (H \times K) \to \mathcal{P}^*(H \times K)$$

by

$$(a,b) \circ_d (c,d) := (a \cdot_H c) \times (b \cdot_K d).$$

For all $a, c \in H$ and $b, d \in K$.

And we obtain ;

$$(A \times B) *_d (C \times D) := (A *_H C) \times (B *_K D).$$

For any $A \times B, C \times D \in \mathcal{P}^*(H \times K)$ such that $*_H$ is a operation on $\mathcal{P}^*(H)$ and $*_K$ is a operation on $\mathcal{P}^*(K)$.

Furthermore ;

$$\{(a,b)\} *_d [(c,d) \circ_d (e,f)] := [\{a\} *_H (c \cdot_H e)] \times [\{b\} *_K (d \cdot_K f)].$$

and

$$[(a,b) \circ_d [(c,d)] *_d \{(e,f)\} := [(a \cdot_H c) *_H \{e\}] \times [(b \cdot_K d) *_K \{f\}].$$

For any $a, c, e \in H$ and $b, d, f \in K$.

From 3., the following results are derived:

3.1) Let H and K be hypersemigroups. Then $H \times K$ is a hypersemigroup.

3.2) Let A be a subhypersemigroup of a hypersemigroup H and B be a subhypersemigroup of a hypersemigroup K. Then $A \times B$ is a subhypersemigroup of $H \times K$.

3.3) Let B be a bi-interior hyperideal of H and C be a bi-interior hyperideal of K. Then $B \times C$ is a bi-interior hyperideal of $H \times K$.

4. Let H be a hypersemigroup. Then the following statements hold.

   (1) Every left hyperideal of H is a bi-interior hyperideal of H.

   (2) Every right hyperideal of H is a bi-interior hyperideal of H.

   (3) Every two-side hyperideal of H is a bi-interior hyperideal of H.

5. The intersection of a right and a left hyperideal of H, if it is a non-empty set, then it is a bi-interior hyperideal of H.

6. Let H be a hypersemigroup. Then every quasi-hyperideal of H is a bi-interior hyperideal of H.

7. Let H be a hypersemigroup. Then every bi-hyperideal of H is a bi-interior hyperideal of H.

8. Let H be a hypersemigroup with the identity e. Then,

$$A * \{e\} = A = \{e\} * A.$$

   for all non-empty subset A of H.

9. Let H be a hypersemigroup. Then every interior hyperideal of H is a bi-interior hyperideal of H.

10. Let H be a hypersemigroup with identity e. Suppose that $H = H * \{a\}$ for all $a \in H$. Then every bi-interior hyperideal of H is a quasi-hyperideal of H.

11. Let H be a hypersemigroup with the identity e. Then every bi-interior hyperideal of H is an interior hyperideal of H.

12. Let H be a hypersemigroup. Then the following statements hold.

   (1) If H is left simple, then every bi-interior hyperideal of H is a right hyperideal of H.

   (2) If H is right simple, then every bi-interior hyperideal of H is a left hyperideal of H.

   (3) If H is simple, then every bi-interior hyperideal of H is a two-side hyperideal of H.

13. Let H be a simple hypersemigroup. Then every bi-interior hyperideal of H is a bi-hyperideal of H.

14. Let H be a regular hypersemigroup. Then $B \subseteq (H * B * H) \cap (B * H * B)$ for any subset B of H.

15. Let H be a regular hypersemigroup. Then the following statements are equivalent.
    (1) B is a bi-interior hyperideal of H.
    (2) $B = R * L$ for some right hyperideal R and left hyperideal L of H.

16. Let H be a hypersemigroup. Then the following conditions are equivalent.
    (1) H is regular.
    (2) $A = (A * H * A) \cap (H * A * H)$ for every bi-interior hyperideal A of H.
    (3) $A \cap L \subseteq A * L$ for every bi-interior hyperideal A and every left hyperideal L of H.
    (4) $R \cap A \subseteq R * A$ for every right hyperideal R and every bi-interior hyperideal A of H.

17. Let H be a hypersemigroup. Suppose that R and L is a minimal right and a minimal left hyperideal of H, respectively. We have that $R * L$ is a minimal bi-interior hyperideal of H.

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